PHY311 : Classical Mechanics

Assignment - 2, 15.9.2016

Note 1 : This collection of problems have been taken from various sources; text books and other internet sources. You are encouraged to try these problems and many others you might find elsewhere.

Note 2 : This assignment is not meant for evaluation. You should attempt to solve these problems but you need not submit it back.

1. A dynamical system has the Lagrangian

$$L = \frac{p^2}{2\alpha} - bqpe^{-\alpha t} + \frac{ba}{2}q^2 e^{-\alpha t}(\alpha + be^{-\alpha t}) + \frac{kq^2}{2}$$

where $, b, k_1$ and k_2 are constants. Find the equations of motion in the Hamiltonian formulation.

2. Obtain the Lagrange equations of motion for a spherical pendulum, i.e., a mass point suspended by a rigid weightless rod. Use the Lagrangian to obtain the Hamiltonian for this system.

3. Obtain the Hamiltonian and equations of motion for the double pendulum. The first pendulum has length L_1 with mass m_1 suspended from it. The second pendulum has length L_2 with mass m_2 and it is suspended from the lowest point of the first pendulum.

4. A particle of mass m can move in one dimension underr the influence of two springs connected to fixed points distance a apart (See the reference figure in problem 16, Chapter 8 of GoldStein's book). The springs obey Hooke's law and have force constants k_1 and k_2. Using position of the particle from one fixed as the generalised coordinate, obtain the Hamiltonian for this system. Verify if the energy and the Hamiltonian are conserved.

5. The Lagrangian for a one-degree of freedom system can be written as,

$$L = \frac{m}{2}\left(\dot{q}^2 \sin^2 \omega t + \dot{q}q\omega \sin 2\omega t + q^2\omega^2\right)$$

Find the Hamiltonian and check if it is conserved.

6. One way of showing that a transformation is canonical is by showing that the fundamental Poisson brackets are invariant when evaluated with respect to the old and new coordinates, that is, $\{Q, P\}_{q,p} = \{Q, P\}_{Q,P} = 1$.
Show that the transformation represented by

$$Q = \ln(1 + \sqrt{q}\cos p), \qquad P = 2(1 + \sqrt{q}\cos p)\sqrt{q}\sin p$$

is canonical. Show that $F(p, Q) = -(e^Q - 1)^2 \tan p$ is the generating function for this transformation.

7. For what values of α and β do the equations

$$Q = q^{\alpha} \cos(\beta p), \qquad P = q^{\alpha} \sin(\beta p)$$

represent a canonical transformation. Find the generating function as well.

8. A block of mass m is attached to the end of a spring of spring constant k. This whole system is placed in massless cart which is moving with a uniform velocity v. Find the time-dependent Lagrangian $L(q, \dot{q}, t)$ for this system. Find the corresponding Hamiltonian. In this case, you will get a time-dependent Hamiltonian.
Now perform a coordinate transformation to absorb this time dependence and obtain a time-independent Hamiltonian for this system.
(Hint : Change to new coordinates represented by $Q = q - vt$, where q is the old generalised coordinate.)

9. For a system with 3 degrees of freedom, consider the following functions,

$$L_1 = p_2 q_3 p_3 q_2, \qquad L_2 = p_3 q_1 p_1 q_3.$$

Show that $\{L_1, H\} = 0$ and $\{L_2, H\} = 0$ if the Hamiltonian is given by,

$$H = \frac{|\mathbf{p}|^2}{2m} + V(|\mathbf{q}|).$$

Show also that $\{-\{L_1, L_2\}, H\} = 0$.

10. If a dynamical variable D is such that $\{D, H\} = 0$, then D is a conserved quantity. For the Hamiltonian given by,

$$H = \frac{p^2}{2} - \frac{1}{2q^2},$$

show that $D = pq/2 - Ht$ is a constant of motion by evaluating the appropriate Poisson bracket.

11. Do problems 4, 5, 6, 8, 16, 17 and 18 in chapter 9 in the book 'Classical Mechanics' by H. Goldstein.

12. A particle with mass m, position $\mathbf{x}$ and momentum $\mathbf{p}$ has angular momentum $\mathbf{L} = \mathbf{x} \times \mathbf{p}$. Evaluate $\{x_j, L_k\}$, $\{p_j, L_k\}$ and $\{L_j, L_k\}$.