PHY311 : Classical Mechanics

Assignment - 1, 1.9.2016

Note 1 : This collection of problems have been taken from various sources; text books and other internet sources. You are encouraged to try these problems and many others you might find elsewhere.

Note 2 : This assignment is not meant for evaluation. You should attempt to solve these problems but you need not submit it back.

1. Show the a Lagrangian $L(q, \dot{q}, t)$ is invariant under a point transformation, i.e. under the transformation $q \to q(x_1, x_2, \ldots x_n; t)$.

2. Obtain the Lagrange equations of motion for a spherical pendulum, i.e., a mass point suspended by a rigid weightless rod.

3. Obtain the Lagrangian and equations of motion for the double pendulum. The first pendulum has length L_1 with mass m_1 suspended from it. The second pendulum has length L_2 with mass m_2 and it is suspended from the lowest point of the first pendulum.

5. Consider central forces in 2D. That is, the potential is $V(r)$, where r is radial distance of the particle of mass m from centre. Set up Lagrangian for this system and show obtain the constants of motion.

5. A block of mass M is constrained to move in a straight line on a frictionless surface with a simple pendulum of length r and mass m attached to it. Write down the Lagrangian for this system in generalised coordinates and obtain the equations of motion.

6. Obtain the equation of motion for the standard harmonic oscillator problem using the principle of virtual work.

7. Consider the Atwood's machine problem for which the details are given in *Classical Mechanics* by H. Goldstein. Use the Lagrangian formalism to obtain the equations of motion and solve it.

8. Show that if the system is invariant under infinitesimal rotation, then the angular momentum L is conserved.

9. A point mass is constrained to move on a massless hoop of radius a fixed in a vertical plane that is rotating about the vertical with contant angular speed ω. Obtain the Lagrange's equations of motion assuming the only external forces arise from gravity. What are the constants of motion.

10. Consider two masses m_1 and m_2 moving in 3D and attracted to each other gravitationally, and are also acted by a uniform gravitational field with acceleration strenght g in the $-\hat{z}$ direction. Write down the Lagrangian for this system. Use the centre of mass coordinates and the vector distance $\vec{r}$ between the two particles.

11. A massless spring has an unstretched length b and spring constant k, and is used to connect two particles of mass m_1 and m_2. The system rests on a frictionless table and may oscillate and rotate. What is the Lagrangian written in some suitable generalised coordinates. Identify the cyclic coordinates and cooresponding constants of motion.