Problem sheet : 4

PHY 310; Mathematical Methods.
Need not be submitted back. Not for evaluation.

1. Obtain the Fourier coefficients of $f(x) = x$ in $0 \leq x \leq \pi$.

2. Obtain the Fourier coefficients of $f(x) = x$ in $-\pi \leq x \leq \pi$. Show that $\sum_{n=1}^{\infty} \frac{-1^{n+1}}{n^2} = \frac{\pi}{12}$.

3. A rectangular pulse is given by,

$$f(x) = 1, \quad |x| < a, \qquad\qquad f(x) = 0, \quad |x| > a.$$

Show that the Fourier exponential transform is $F\omega) = \sqrt{2/\pi}\frac{\sin a\omega}{\omega}$.

4. Show that the Fourier sine transform of $1/\sqrt{x}$ is $\sqrt{\omega}$.

5. Using the properties of delta function, show that $\delta[a(x - x_0)] = \frac{1}{a}\delta(x - x_0)$.

6. Let $f(x) = x$ in $-1 < x < 1$ and its Fourier representation is given by,

$$f(x) = \frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin n\pi x}{n}$$

Use this to show that $\sum_{n=1}^{\infty} 1/n^2 = \pi^2/6$.

7. Find the Fourier series for $f(x) = |x|$ in $-\pi/2 < x < \pi/2$. Also use this to sum the series $\sum_{\text{odd}_n} 1/n^4$.

8. Use Parseval's relation to evaluate

$$\int_{-\infty}^{\infty} \frac{dk}{(k^2 + a^2)^2} = \frac{\pi}{2a^3}.$$

9. Solve $dy/dx + \alpha x = 0$ using Fourier transforms.

10. Show that Laplace transform of $\delta(t)$ is unity. What is the Laplace transform of $\delta(t - t_0)$

11. Using Laplace transforms, solve m X"(t) + b X'(t) + k X(t) = 0. (See Example 15.10.1 in Arfken and Weber).

12. For unit step function, $\theta(t - k)$, verify that its Laplace tranform is e^{ks}/s.

13. Calculate the inverse Laplace tranform for the following ;
(a) $1/(s^2 + a^2)^2$
(b) $1/s^2(s^2 + a^2)^2$.
(c) $s/(s^2 + a^2)^2$

14. Show that $e^{-bs}f(s) = \mathcal{L}[F(t - b)]$.

15. Show that the Laplace transform of the square wave of period T given by,

$$F(t) = 1 \quad \text{for} \quad 0 < t < T/2$$

$$F(t) = 0 \quad \text{for} \quad T/2 < t < T$$

is $(1/s)(1 - e^{as/2})/(1 - e^{as})$.

16. Show that the Dirac delta function $\delta(x-a)$ expanded in a Fourier sine series in the half-interval $0 < a < L$ is given by,

$$\delta(x-a) = \frac{2}{L}\sum_{n=1}^{\infty} \sin\left(\frac{n\pi a}{L}\right)\sin\left(\frac{n\pi x}{L}\right).$$

17. Expand $f(x) = x$ in the interval $(0, 2L)$ and sketch the first three terms of the series.