Problem sheet : 3

PHY 310; Mathematical Methods.

1. Determine the location and the nature of singularity (regular or irregular) of the following differential equations. Include the point at infinity too.
(a) $xy'' + (1-x)y' + ay = 0$
(b) $y'' - 2xy' + 2ay = 0$

2. Find the general solutions of the following differential equations;
(a) $xy' + (1+x)y = e^x$
(b) $y' = \frac{a^2}{(x+y)^2}$
(c) $y' + y\cos(x) = \frac{1}{2}\sin 2x$

3. Find the general solutions of the following differential equations;
(a) $y'' + 3y' + 2y = e^x$
(b) $y'' + 3y' - 10y = 6e^{4x}$
(c) $y'' + k^2 y = \sin bx (k, b > 0)$

4. Use the method of series solution to solve the following equation.

$$(1-x^2)y'' - xy' + n^2 y$$

This is called Chebyshev equation. Analyse the equation for singularities, obtain the indicial equation and the linearly independent solutions.

5. Using the power series of the form

$$y(x) = \sum_n a_n x^{n+m}$$

solve the following equation.

$$y'' + \omega^2 y = 0$$

Obtain the indicial equation.

6. Test the following equation for exactness and if so, obtain the solution.
$e^y\ dx + (xe^y + 2y)dy = 0.$

7. Given that one solution of $y'' + \frac{y'}{x} - \frac{m^2}{x^2}y = 0$ is $y = x^m$, find the second solution. Verify that both the solutions are linearly independent.

8. Show that $y'' + \frac{1-\alpha^2}{4x^2}y = 0$ has the solutions $y_1 = a_0 x^{(1+\alpha)/2}$ and $y_2 = a_0 x^{(1-\alpha)/2}$.

9. Use power series method to obtain solutions for (a) Laguerre, (b) Hermite (c) Legendre and (c) Confluent hypergeometric equations. Look up "mathematical methods for physicists" by Arfken and Weber for the equations. Check for convergence of solutions.