Mathematical Methods

Assignment : 2

This entire assignment is optional. Does not carry any marks.

1. Consider the following matrix A.

$$\mathbf{A} = \begin{pmatrix} -1 & 1 & 3 \\ 2 & 1 & 1 \\ 4 & 2 & 2 \end{pmatrix}$$

Calculate (i) the transpose (ii) Inverse (iii) determinant and (iv) rank of A.

2. Find the echelon form and hence the inverse of the following matrix.

$$\mathbf{A} = \begin{pmatrix} 2 & 5 \\ 3 & -2 \end{pmatrix}$$

3. Determine if the system of equations has a solution and if so find them using *matrix methods* :

$$x_1 + 3x_2 + x_3 = 4 \ ; \qquad x_1 + x_2 - x_3 = 1 \ ; \qquad 2x_1 + 4x_2 = 5$$

4. Given the vectors $a_1 = \{1, -1, 1\}, a_2 = \{-1, 2, 2\}, a_3 = \{-3, 5, 6\}$, can the vector $\beta = \{-6, 10, 15\}$ be a point in the vector space given by a_1, a_2, a_3.

5. Consider the matrix,

$$\mathbf{A} = \begin{pmatrix} 1 & 3 & 1 \\ 3 & 1 & 2 \\ 1 & 2 & 5 \end{pmatrix}$$

Find its characteristic polynomial, eigenvalues and the normalised eigenvectors.

6. Consider the matrix,

$$\mathbf{A} = \begin{pmatrix} 2 & 2 & 1 \\ 3 & 1 & 4 \\ 7 & 2 & 5 \end{pmatrix}$$

Find its eigenvalues and eigenvectors.

7. Show that if A is a symmetric matrix, then A^{-1} is also symmetric if it exits.

8. For the matrix in problem 2, orthonormalise the vectors using Gram-Schmidt procedure.

9. Show that for upper (lower) triangular matrices, the eigenvalues are the diagonal entries.

10. For the matrix in problem 2, calculate $\exp(A)$.

11. A matrix A is said to be idempotent if $A^2 = A$. Show that the only possible eigenvalues of idempotent matrix are zero and one.

12. Show that the rotation matrix in 3-dimensional space is orthogonal.

13. Show that the eigenvalues of the unitary matrices are of unit magnitude. Show that it is also true for orthogonal matrix.

14. Find the (a) rank, (b) dimension of null space, (b) basis for null space and (iv) the left or right inverse (when they exist) for

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$$

15. Find the rank, nullity and the basis for null space of

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 4 & 0 \\ 0 & 2 & 8 & 0 \end{pmatrix}$$

16. Show that the trace of a real symmetric matrix remains invariant under an orthogonal transformation.

17. Show that similar matrices have the same set of eigenvalues. Do they have the same set of eigenvectors. What can be said about the eigenvectors ?

18. Find the geometric and algebraic multiplicity of the eigenvalues of

$$\mathbf{A} = \begin{pmatrix} 5 & 4 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ -1 & -1 & 3 & 0 \\ 1 & 1 & -1 & 2 \end{pmatrix}$$

Is this matrix diagonalisable ? If not, find the Jordan normal form.

19. Find the eigenvalues and their geometric and algebraic multiplicity for

$$\mathbf{A} = \begin{pmatrix} -1 & 3 & 0 \\ 4 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

Find the basis vectors that lead to either diagonal or Jordan normal form.