Mathematical Methods

Assignment - 1

Problems 3 to 6 and 8 are optional. Due on or before 10/9/2009

1. Obtain the Taylor expansion :
(a) $1/(1-z^2)$ in $|z|<1$
(b) $\sin z/z$ in $0<|z|<\infty$
(c) $(\cos z-1)/z^2 \quad \text{in} \quad 0<|z|<\infty$
(d) $\ln(1+z) \quad \text{for} \quad |z|<1$

2. Obtain the Laurent series for

$$f(z)=\frac{z}{(z-2)(z+i)}$$

in (a) $|z|<1$, (b) $1<|z|<2$, (c) $|z|>2$

3. Expand $f(z)=z/(z^2-a^2)$ $(a>0)$ in a Laurent series about $z=0$ in the region $|z|<a$ and $|z|>a$.

4. Obtain the Laurent series for

$$f(z)=\frac{1}{z(z^2+1)}$$

in (a) $0<|z|<1$, (b) $1<|z|<2$

5. Evaluate the integrals on a contour C of unit circle centred at the origin.
(a) $\exp(z)/z^3$
(b) $1/(z^2\sin z)$
(c) $\exp(1/z)$

6. Show using Cauchy residue theorem,

$$\int_{|z|=2}\frac{e^{iz}}{z^4+2z^2+1}\,dz \quad =\frac{\pi}{e}$$

7. Evaluate

$$\int_0^\infty \frac{\cos kx}{1+x^4}\,dx$$

8. Determine the type of singularity and classify the type of pole, if any. (include the point at $z=\infty$).
(a) $ze^{iz}/(z^2+a^2)$, $(a>0)$
(b) $z^{-k}/(1+z) \quad \text{where} \quad 0<k<1$
(c) $z^2e^z/(1+e^{2z})$
(d) $1/(e^{z^2}-1)$

9. Do the following problems from Arfken and Weber (sixth edition, 2005) (pages 474-480) :
7.1.8, 7.1.9, 7.1.10, 7.1.11, 7.1.12, 7.1.15, 7.1.18, 7.1.20, 7.1.21