PHY102 : Waves and Matter

Problem sheet - 5

Optional. You do not have to submit it.
You are encouraged to try out more problems than what is given here.

1. Do the following problems from the book "The physics of vibrations and waves" (6th edition) by H. J. Pain. Problems 5.4, 5.5, 5.7, 5.8, 5.12, 5.14, 5.15, 5.17.

2. At $t = 0$, a progressive wave pulse is given by $y = 10/(x^2 + 5)$. What will be function representing the pulse at time t, if the pulse is travelling along negative x-direction with speed 5 m/sec.

3. A standing wave is formed on a string by two counter-propogating waves of the form, $y_1 = a\sin(kx - \omega t)$ and $y_2 = a\sin(kx + \omega t)$ travelling in opposite directions. If the mass density of string is μ and its cross-sectional area is A, show that the total energy between two adjacent nodes on this string is $\pi\mu a^2\omega^2 s/k$.

4. A plane progressive wave with amplitude 1 cm is generated using a tuning fork at one end ($x = 0$) of a long string. The displacements of the particles at $x = 10$ cm and at $x = 20$ cm, at some time instants, are -0.5 cm and 0.5 cm, respectively. The speed of the wave is 100 m/s. Calculate the frequency of the tuning fork ?
If the wave is travelling along the positive x-direction and at $t = 0$, the displacement at $x = 0$ is at equilibrium position, then write the explicit form of displacement in terms of amplitude, frequency and the wavelength.

5. A plane progressive wave of amplitude 0.02 cm is generated by a musical instrument. If a note of frequency 300 Hz is produced, calculate the rate at which the energy is generated per unit volume, if the density of air is 1.29 kg/m^3. Note that energy generated per unit volume is the energy density.

6. Show that the average kinetic energy transported by a progressive wave per cycle through a thin layer of mass dm is equal to the average potential energy of the mass dm of a layer in the medium.

7. A stretched string of mass m, length L and tension T is driven by two sources, one at each end. The sources both have the same frequency ν and amplitude A, but are exactly 180^o out of phase with one another. What is the smallest possible value of ω consistent with normal mode vibrations of the string.

8. A wire of length L and mass m has variable linear density $\rho = kx$ where k is a constant and x is the distance from one end of a string. Assuming that the wire is under tension T, show that the time taken for a pulse to travel from one end of wire to the other end is

$(2/3)\sqrt{2mL/T}$.

9. Consider a system of N coupled oscillators driven at a frequency $\omega < 2\omega_0$ (i.e., $Y_0 = 0, Y_{N+1} = h\cos\omega t$). Find the resulting amplitudes of the N oscillators. [Hint: The differential equations of motion are the same as in the undriven case (only the boundary conditions are different). Hence try $A_p = C\sin\alpha p$, and determine the necessary values of α and C.

10. From the book 'Vibration and waves' by A. P. French do the following problems; Problems 7.1 to 7.5 and 7.14, 7.15 and 7.16 (in pages 246 to 249).

(Some of these problems are based on the problems given in 'Vibration and waves' by A. P. French).