PHY102 : Waves and Matter

Assignment - 3

Optional. You do not have to submit it.
You are encouraged to try out all the problems.

1. Do the following problems from the book "The physics of vibrations and waves" (6th edition) by H. J. Pain. Problems 3.1, 3.2, 3.3, 3.4, 3.6, 3.11.

2. This is a problem on simple harmonic motion without damping or driving. A block on a spring oscillates with a period of 0.8 sec and its amplitude is 10 cm. At initial time, the block is left to the equilibrium and is moving towards the left. What is its position and direction of motion at time $t = 2$ sec.

3. A mass of 2 kg oscillates on a spring with constant 50 N/m. By what factor does the frequency of oscillation decrease when a damping force with constant $\gamma = 12$ is introduced ?

4. The amplitude of a lightly damped oscillator decreases by 5% during each cycle. What percentage of the mechanical energy of the oscillator is lost in each cycle?

5. Consider the damped and forced harmonic oscillator given by,

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 10y = 0, \qquad y(0) = -1/2, \dot{y}(0) = 4.0.$$

Find the solution $y(t)$ and verify that it satisfies the differential equation.

6. Consider a damped oscillator with $k = 32$, $m = 0.5$, $\gamma = 1$ in MKS units. *(i)* Find the solution with initial conditions $x(0) = 2$, $v(0) = 0$. *(ii)* If, in addition a driving force $F_0 \cos \omega t$ with $F_0 = 10N$ and $\omega = 2\omega_0$, then find the solution with $x(0) = 2, v(0) = 0$.

7. Consider a damped oscillator without any drive. Find the time after which the ampitude of oscillation drops to half its initial value.

8. A simple pendulum has a length (l) of 1 m. In free vibration the amplitude of its swings falls off by a factor e in 50 swings. The pendu- lum is set into forced vibration by moving its point of suspension horizontally in SHM with an amplitude of 1 mm.
(a) Show that if the horizontal displacement of the pendulum bob is x, and the horizontal displacement of the support is u, the equation of motion of the bob for small oscillations is

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \frac{g}{l}x = \frac{g}{l}u$$

Solve this equation for steady-state motion, if $u = u_0 \cos \omega t$. (Put $\omega_0^2 = g/l$.)
(b) At exact resonance, what is the amplitude of the motion of the pendulum bob? (First, use the given information to find Q.)
(c) At what angular frequencies is the amplitude half of its resonant value ?

9. Consider a system with a damping force undergoing forced oscillations at an angular frequency ω.
(a) What is the instantaneous kinetic energy of the system?
(b) What is the instantaneous potential energy of the system?
(c) What is the ratio of the average kinetic energy to the average potential energy? Express the answer in terms of the ratio ω/ω_0.
(d) For what value(s) of ω are the average kinetic energy and the average potential energy equal? What is the total energy of the system under these conditions ?
(e) How does the total energy of the system vary with time for an arbitrary value of ω ? For what value(s) of ω is the total energy constant in time ?

10. A mass m is subject to a resistive force $-bv$ but no springlike restoring force.
(a) Show that its displacement as a function of time is of the form

$$x(t) = C - \frac{v_0}{\gamma} e^{-\gamma t}$$

where $\gamma = b/m$.
(b) At $t = 0$ the mass is at rest at $x = O$. At this instant a driving force $F = F_0 \cos \omega t$ is switched on. Find the values of A and δ in the steady-state solution $x = A \cos(\omega t - \delta)$.
(c) Write down the general solution [the sum of parts (a) and (b)] and find the values of C and v_0 from the conditions that $x = 0$ and $dx/dt = 0$ at $t = O$. Sketch x as a function of t.

11. An object of mass 2 kg hangs from a spring of negligible mass. The spring is extended by 2.5 cm when the object is attached. The top end of the spring is oscillated up and down in SHM with an amplitude of 1 mm. The Q value of the system is 15. (a) What is ω_0 for this system ?
(b) What is the amplitude of forced oscillation at $\omega = \omega_0$?

(Note : Some problems here are taken from the book 'Vibrations and waves' by A. P. French.)