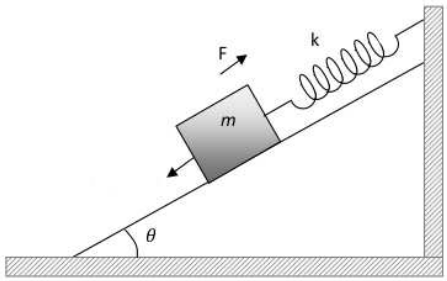
PHY102 : Waves and Matter

Problem Sheet - 2

This is not for evaluation. You do not have to submit it.
However, you are strongly encouraged to try out all the problems.

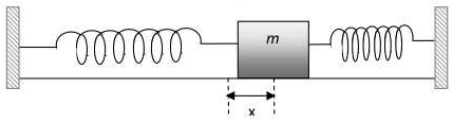
1. A block of mass 0.2 kg slides on an inclined plane which makes an angle $\theta = 30^o$ with the horizontal. Assume the plane to be friction-less. The block is connected through a spring to the top of the plane using a spring of spring constant 80 N/m, as shown in the figure below. The length of the spring in its relaxed state is 23.75 cm.



How far from the top of the incline does the block stop ? If the block is pulled and released, find an expression for the time period of oscillation. Use the given numbers to determine the time period ?

2. A spring whose length is L and spring constant is k is cut in to two pieces of equal length. What is the spring constant of each of the new springs ?

3. See the configuration of two springs and a block shown in the figure below. The left and right springs have spring constant k_l and k_r. The block has mass M.



Derive an expression for the time period of oscillations of the block.

4. A particle is executing simple harmonic oscillation. At two instants of time, (say at t_1 and t_2) the distance of the particle from the equilibrium position is x_1 and x_2. The corresponding values of velocity are v_1 and v_2. Then, show that the time period is given by

$$T = 2\pi \left(\frac{x_2^2 - x_1^2}{v_2^2 - v_1^2} \right).$$

5. A car is moving on a horizontal road with uniform acceleration a. A simple pendulum is suspended from the ceiling of this car. If the pendulum has length L, find the time period of small oscillations about its equilibrium position.

6. A tunnel is dug between two diametrically opposite points of the earth. Treat the earth as a solid sphere of uniform density ρ. Show that if a particle is released into this tunnel, it will execute a simple harmonic motion. Find the time period of its motion.

7. A harmonic oscillator consists of a block hanging from a spring. The block is immersed in a viscous liquid. Assume that a damping force $F = -\gamma v$ will come into play. In this, γ is the damping coefficient and v is the velocity. Initially, time $t = 0$ it oscillates with an amplitude of 25 cm. The amplitude falls to 75% of this initial value at the completion of four oscillations.
(a) Find the value of γ,
(b) Determine the total energy dissipated at the end of four oscillations.

8. Do problems 2.1, 2.2, 2.3, 2.5 from the book *The Physics of Vibrations and Waves* by H. J. Pain.

9. Two vibrations perpendicular to one another are described by $x(t) = 7\cos(5\pi t)$ and $y(t) = 7\cos(10\pi t + \pi/3)$. Construct Lissajous figure for the combined motion.

10. Find the frequency of the combined motion for the following oscillatory solutions;
(a) $\sin(2\pi t - \sqrt{2}) + \cos(2\pi t)$,
(b) $\sin(12\pi t) + \cos(13\pi t - \pi/4)$.

11. Two oscillations along the same line are described by the equations,

$$u_1 = A\cos 10\pi t, \qquad u_2 = A\cos 12\pi t$$

Find the beat period, and sketch the resultant oscillatory behaviour for one beat period.