(b) Linear combination of Solutions is also a solution.

If $A \sin \omega t$ and $B \cos \omega t$ are the two independent solutions, then $A \sin \omega t + B \cos \omega t$ is also a valid solution.

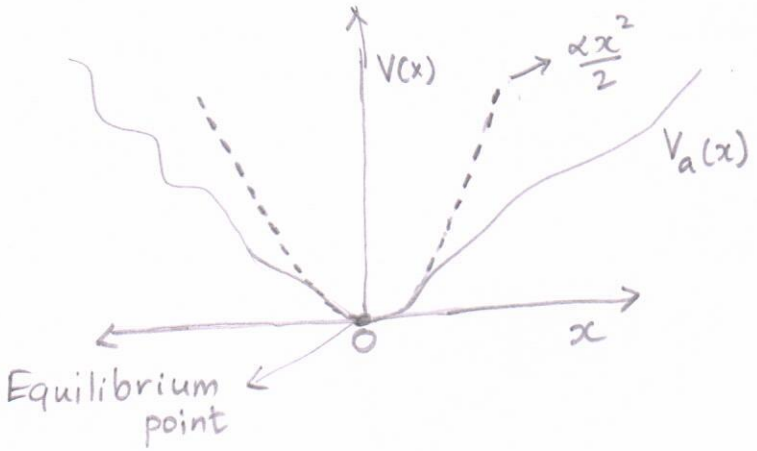
Total energy $E = KE + PE = \frac{1}{2} m \dot{x}^2 + \frac{\alpha x^2}{2}$ (2)

$PE = \frac{\alpha x^2}{2}$ (can be calculated from restoring force $= -\alpha x$)

$KE = \frac{1}{2} m \dot{x}^2$.

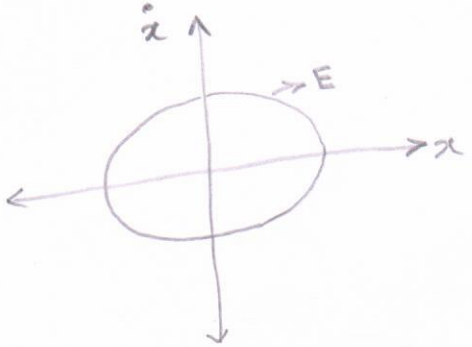
From Eqn (2), $\frac{dE}{dt} = 0$. Implies Energy is constant of motion.

If $V_a(x)$ is any arbitrary (nonlinear) smooth potential function, then the dynamics around its equilibrium points is harmonic oscillation. Can be described by Eqn. (1).



Since E is a constant, eqn (2) represents an ellipse in $x - \dot{x}$ Space.

Each ellipse is characterised by a value of E.



"State Space" view.