Q-factor of the oscillator

This is a measure how much energy is lost in one complete cycle of oscillation.

For dissipative harmonic oscillator, energy decays as

$$E(t) = E_0 e^{-2pt} = E_0 e^{-\frac{\gamma t}{m}}$$

Quality factor is defined as the dimensionless ratio

$$Q = \frac{\text{Energy in the oscillator}}{\text{Energy lost in one cycle}}$$

For dissipative oscillator, time period (or one cycle) is $T = 2\pi/\omega$ (ω: angular frequency)

Energy at $t = \tau_0$: $E(\tau_0) = E_0 e^{-\gamma \tau_0/m}$

Energy lost in one cycle : $E(\tau_0) - E(\tau_0 + 2\pi/\omega)$

$$\therefore Q = \frac{E_0 e^{-\gamma \tau_0/m}}{E_0 e^{-\gamma \tau_0/m} - E_0 e^{-\gamma(\tau_0 + 2\pi/\omega)/m}} = \frac{1}{1 - e^{-2\pi\gamma/m\omega}}$$

If dissipation is weak, i.e, $\gamma \to 0$. Then $1 - e^{-2\pi\gamma/m\omega} \simeq \frac{2\pi\gamma}{m\omega}$.

$$\therefore \frac{Q}{2\pi} = \frac{m\omega}{2\pi\gamma} = \frac{m\, 2\pi\nu}{2\pi\gamma} = \frac{m\nu}{\gamma}$$

(Note that Q-factor is usually defined with 2π factor in the denominator.)

ν is the frequency of the oscillator.