PHY103 Quiz-3 Brief Solutions

1) Start from $PV^\gamma = \text{constant}$. Taking $\log_e$ both sides, we get $\log P + \gamma \log V = 0$

Take differential to get $\frac{dP}{P} + \gamma \frac{dV}{V} = 0$

This gives, Bulk modulus $B = \frac{-dP}{(dV/V)} = \gamma P$.

2) For ideal gases, $PV = nRT$, or $P = \rho R_s T$.

Speed of sound $v = \sqrt{\frac{\gamma P}{\rho}} \Rightarrow v^2 = \gamma R_s T$

R_s → Specific gas constant.

$\therefore \frac{v_1^2}{v_2^2} = \frac{T_1}{T_2}$

3) Figure (drawn by hand) has imperfections.

From the figure, $y(x) = \frac{b+a}{2} + \left(\frac{b-a}{2}\right) \sin(x-\pi)$

for any real values of a and b.

$\therefore \langle y \rangle = \frac{b+a}{2}$.

Since $\sin(x-\pi) = -\sin x$, $y(x) = \frac{b+a}{2} - \left(\frac{b-a}{2}\right) \sin x$.

This is already in Fourier series form. It has only one Fourier coefficient : $-\left(\frac{b-a}{2}\right)$.

4) Since this behaves exactly like a string tied between two walls, we have for 3rd harmonic, $L = \frac{3}{2}\lambda$.

L is the length of the pipe.

From $P(x,t) = 2.5 \sin\left(\frac{\pi}{2}x - \frac{300\pi}{2}t\right)$, we get $\lambda = \frac{2\pi}{k} = 4$.

$\therefore L = \frac{3}{2} \times 4 = 6$ m.

5) See problem sheet 6 for a complete solution.

$\cos^2 x = a_0 + a_1 \cos 2x$. $a_0 = \frac{1}{2}$ and $a_1 = \frac{1}{2}$.