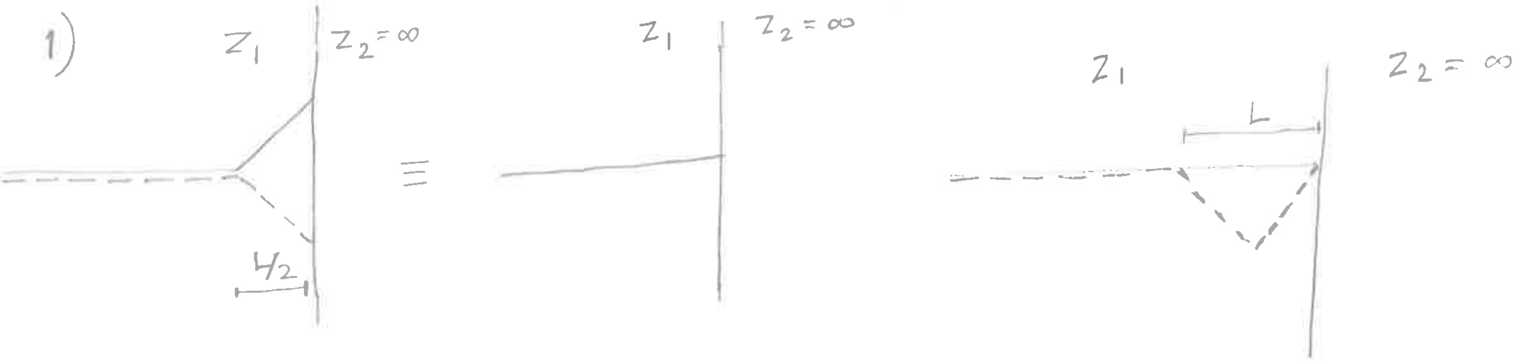
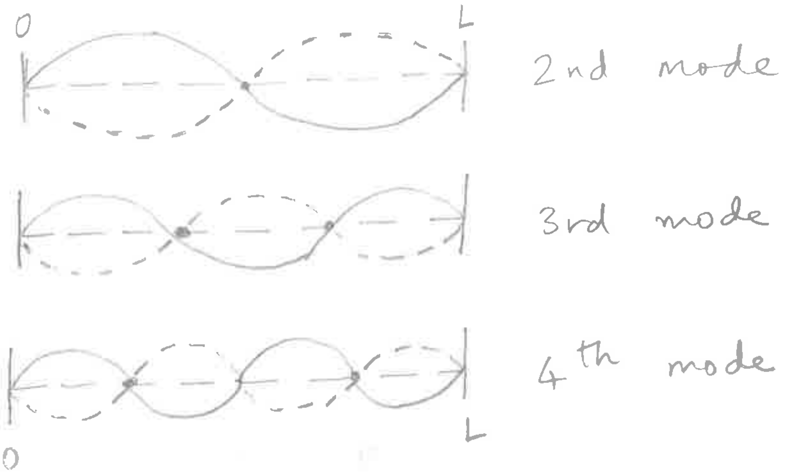
PHY 102 Quiz-2 Brief Solutions

1)



2)



3) group velocity $v_g = v + k\, dv/dk$

$v = c\,\dfrac{\sin(ka/2)}{(ka/2)}$. $\dfrac{dv}{dk} = c\left[\dfrac{\cos(ka/2)}{k} - \dfrac{2\sin(ka/2)}{k^2 a}\right]$

Upon simplifying $v_g = c\cos(ka/2)$

4) a) wave velocity $v = \sqrt{\dfrac{T}{\rho}} = \sqrt{\dfrac{T}{(m/L)}}$ where linear density $\rho = \dfrac{m}{L}$

frequency of fundamental mode $\nu_1 = \dfrac{v}{2L} = \dfrac{1}{2L}\sqrt{\dfrac{TL}{m}}$

$\therefore\ \nu_1 = \dfrac{1}{2}\sqrt{\dfrac{T}{mL}}$

$\nu_1 = \dfrac{1}{2}\sqrt{\dfrac{10}{0.01 \times 2.5}} = \dfrac{20}{2} = 10$ Hz.

b) If touched at a point 0.5m from one end, then only those modes for which $x = 0.5$m is a node will be allowed. Let x be the distance measured from one end.

Then, if Δ represents the distance between nodes, we have $n\Delta = L$, for n^{th} normal mode.

$\therefore$ allowed normal mode will be $n = \frac{L}{\Delta} = \frac{2.5}{0.5} = 5$.

$\therefore$ Allowed frequency will be $5\nu_1 = 50$ Hz.

5) The wave passes from one medium to another. Tension in the string is uniform.

Tension in first string $T_1 = \rho_1 c_1^2$

Tension in second string $T_2 = \rho_2 c_2^2$.

$$T_1 = T_2 \Rightarrow \rho_1 c_1^2 = \rho_2 c_2^2 \Rightarrow \frac{\rho_1}{\rho_2} = \frac{c_2^2}{c_1^2} = \left(\frac{50}{25}\right)^2 = 4$$

Since $c_1 = \frac{\omega_1}{k_1} = \frac{50\pi}{2\pi} = 25$ m/sec. Given that $c_2 = 50$ m/sec.

We know, $\frac{A_r}{A_i} = \frac{Z_1 - Z_2}{Z_1 + Z_2}$ and $\frac{A_t}{A_i} = \frac{2Z_1}{Z_1 + Z_2}$

Then, $$\frac{A_t}{A_i} = \frac{2\rho_1 c_1}{\rho_1 c_1 + \rho_2 c_2} = \frac{2c_1}{c_1 + (\rho_2/\rho_1)c_2} = \frac{2 \times 2.5}{25 + 50/4} = \frac{4}{3}$$

Since $A_i = 2$, $A_t = 8/3$.

Similarly, $A_r = 2/3$. A_r can also be calculated from the relation $A_i + A_r = A_t$.

$$\therefore \quad y_r = \frac{2}{3}\cos(-2\pi x - 50\pi t) = \frac{2}{3}\cos(2\pi x - 50\pi t)$$

$$y_t = \frac{8}{3}\cos(\pi x - 50\pi t)$$

This is because in second string $k_2 = \frac{\omega}{50} = \frac{50\pi}{50} = \pi$

6) Phase difference $\Delta\varphi$ is related to distance Δx as

$$\Delta\varphi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}(x_2 - x_1).$$

For $u(x,t)$: $\Delta\varphi_u = \frac{2\pi}{\lambda_1}(x_2 - x_1) \Rightarrow \lambda_1 = \frac{2\pi}{\Delta\varphi_u}(x_2 - x_1)$

For $v(x,t)$: $\Delta\varphi_v = \frac{2\pi}{\lambda_2}(x_2 - x_1) \Rightarrow \lambda_2 = \frac{2\pi}{\Delta\varphi_v}(x_2 - x_1)$

particle velocity $\frac{du}{dt} = A\omega_1 \cos(\omega_1 t + k_1 x)$

max. particle velocity $\dot{u}_{max} = A\omega_1$

Similarly, for $v(x,t)$, max particle velocity is $\dot{v}_{max} = B\omega_2$

$$\frac{\dot{u}_{max}}{\dot{v}_{max}} = \frac{A\omega_1}{B\omega_2} = \frac{A\, 2\pi c/\lambda_1}{B\, 2\pi c/\lambda_2} =$$

Note the velocity of wave is c in both media.

$$\therefore \frac{\dot{u}_{max}}{\dot{v}_{max}} = \frac{A}{B}\,\frac{\lambda_2}{\lambda_1}$$

$$\frac{\dot{u}_{max}}{\dot{v}_{max}} = \frac{A}{B}\,\frac{(2\pi/\Delta\varphi_v)(x_2 - x_1)}{(2\pi/\Delta\varphi_u)(x_2 - x_1)} = \frac{A}{B}\,\frac{\Delta\varphi_u}{\Delta\varphi_v}$$

$$\therefore \frac{\dot{u}_{max}}{\dot{v}_{max}} = \frac{A}{B}\,\frac{\Delta\varphi_u}{\Delta\varphi_v}$$