Indian Institute of Science Education and Research Pune
Mid-semester Exam, Jan (2019) semester.

Course name: Waves and Matter **Course code:** PHY102
Date: 22.2.2019 (3:00 to 5:00 PM) *Duration:* 2 hours
Instructor : M. S. Santhanam Maximum marks: 50

- This question paper has 5 questions. All of them are compulsory.
- If you draw sketches as an answer, label the axes. No marks without axes labels.
- Show ALL the steps clearly in your calculations while arriving at an answer.
- Unless specified otherwise, all the symbols have their usual meanings.
- Use the same symbols and notation given in the question. Do not use your own.

1.(a) A block attached to a spring oscillates with an amplitude of 10 cm and time period of 2 sec. At time $t = 0$, the block is at $x = 0$. If $x(t) = A\cos(\omega t + \alpha)$, find ω and α.

(b) Displacement of an oscillator is given by $u(t) = a\sin\omega t$. Sketch u vs. du/dt for this system. On the sketch, write the values of points at which your curve crosses the x- and y-axes.

(c) Sketch Lissajous' figure for the following oscillatory motion;
$x(t) = \sin 2\omega t$ and $y(t) = \sqrt{2}\ \sin(\omega t + \pi/4)$. (3+3+4)

2.(a) A particle of mass 0.2 kg executes oscillations. The displacement is given by,
$x(t) = 3\sin(\pi t + \pi/4)$. Find the time instants at which the energy of the particle is purely kinetic.

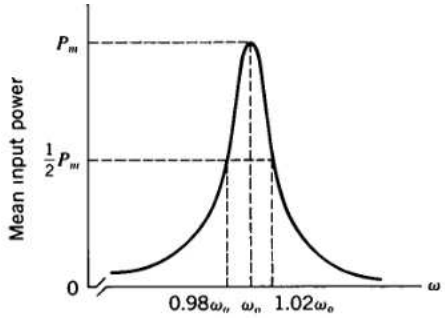
(b) The frequency of a damped simple harmonic oscillator of mass M is given by

$$\omega_d^2 = \omega_0^2 - \frac{\gamma^2}{4M^2}$$

where ω_0 is the natural frequency of the system and γ is dissipation coefficient. Find the value of Q if $\omega_0^2 - \omega_d^2 = 10^{-6}\ \omega_0^2$.

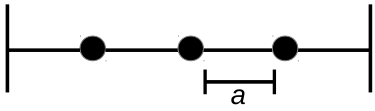
(c) A spring is suspended vertically and a body of mass m hangs from it. This system executes oscillations with angular frequency ω. Now, the oscillations are stopped and the body hangs in equilibrium. Show that the potential energy stored in the spring is $mg^2/2\omega^2$. (3+3+4)

3. The figure below shows the mean power $\bar{P}$ absorbed by a driven and damped oscillating system as a function of driving frequency ω. The driving force is $F_0\sin\omega t$. The mean power input is maximum at $\omega = \omega_0$ and decays to half-maximum at the frequencies $0.98\omega_0$ and $1.02\omega_0$. Answer the following questions;



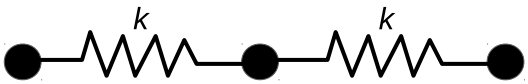
(a) Find the numerical value of Q.
(b) If the driving force is removed, energy decays according to $E(t) = E_0 e^{-\gamma t}$. Find the value of γ in terms of ω_0.
(c) If the driving force is removed, what fraction of energy is lost per cycle.
(d) Physically, what is the information conveyed by the graph displayed above. (3+3+2+2)

4(a) Consider a coupled system shown in the diagram below and give brief answers for the questions that follow. All the blocks are of equal mass.



(i) How many normal modes are there in this system.
(ii) Sketch (do NOT derive) the normal modes with lowest and highest frequency.

(b) Consider the three-block system pictured below;



The three masses are each of mass m and each spring has spring constant k.
(i) Set up the equations of motion for each of the masses.
(ii) Assume oscillatory solution with normal mode frequency ω, and find all possible values of ω.

(4+6)

5.(a) A particle is executing one dimensional simple harmonic motion. When the particle is at distance x_1 and x_2 from equilibrium position, the corresponding values of velocity are v_1 and v_2. Show that the period of oscillation is given by,

$$T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}.$$

(b) A platform is executing simple harmonic motion in a vertical direction with an amplitude of 5 cm and a frequency of $\nu = 10/\pi$ vibrations per second. The displacement of the platform can be written as $x(t) = A\ \sin\omega t$. A block is placed on the platform at the lowest point of its path.
(**NOTE:** Take $g = 10\ m/sec^2$ and $\pi^2 \approx 10$ and you will not need calculator to solve this problem.)
(i) At what point will the block leave the platform ?
(ii) How far will the block rise above the highest point reached by the platform ?

(5+5)