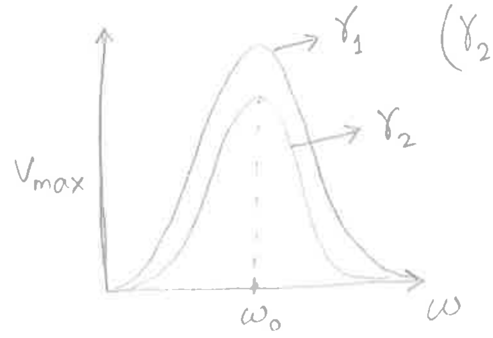
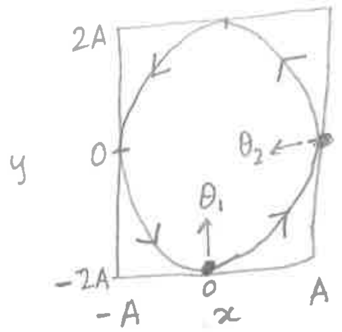
PHY 102 - End Sem Exam - Jan 2019 Sem - Brief Solutions

1(a)



b) $\frac{x}{A} = \sin \omega t$ and $\frac{y}{2A} = -\cos \omega t$. Squaring & adding, we get



Note $\omega = \frac{2\pi}{T}$

$T \to$ time period

c) Put $t=0$. Then, at $t=0$, $x=0$ and $y=-2A$ (point θ_1 in the fig.)

Put $t=\frac{T}{4}$. Then at $t=\frac{T}{4}$, $x=A$ and $y=0$ $\to$ [point θ_2].

This fixes the direction to be anti-clockwise.

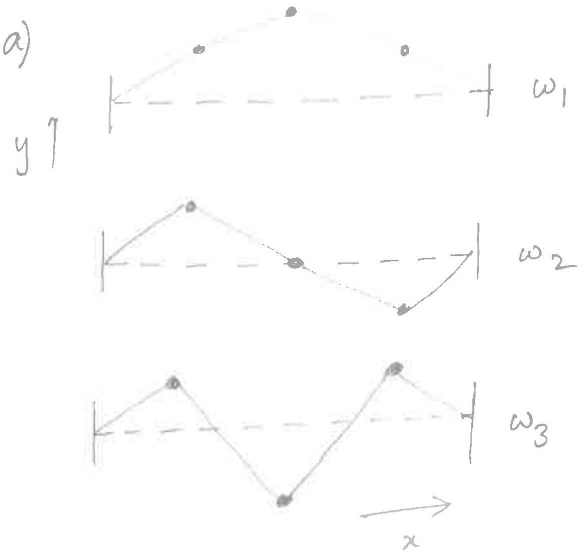
d) $A = A_0\, e^{-\gamma/2m\, t}$ and $\frac{\gamma}{2m} t = 1$ when $t = N T_0$

$\therefore \frac{\gamma}{2m} N T_0 = 1.$

Now, $\frac{T}{T_0} = \frac{2\pi}{\omega_0} \times \frac{\omega_d}{2\pi} = \frac{\omega_d}{\omega_0} = \frac{\sqrt{-\frac{\gamma^2}{4m^2} + \frac{k}{m}}}{\sqrt{k/m}} = \sqrt{1 - \frac{\gamma^2}{4km}}$

$$\frac{T_0}{T_d} = \sqrt{1 - \frac{\gamma^2 m}{4km^2}} = \sqrt{1 - \frac{1}{N^2 T_0^2 \omega_0^2}} = \sqrt{1 - \frac{1}{4\pi^2 N^2}}.$$

2 (a)



$\omega_1 < \omega_2 < \omega_3$

(b) group velocity $V_g = \frac{d\omega}{dk} = k\frac{dV_p}{dk} + V_p$

$$V_g = k\frac{d}{dk}\left(\frac{kS}{\rho}\right)^{1/2} + \left(\frac{kS}{\rho}\right)^{1/2}$$

Upon simplification, this gives $V_g = \frac{3}{2}V_p$

(c) Work done $dW = P\,dt$ = "power × time"

Resistive force $= \gamma v$. Corresponding power : γv^2

$\therefore$ $dW = \gamma v^2\,dt$

Substituting $v(t) = \cos(\omega t - \varphi)$ and integrating over one full time period, we get

$$W = \frac{F_0^2 T}{2\gamma} = \frac{F_0^2}{2\gamma}\cdot\frac{2\pi}{\omega_0}$$

From the given graph, $\overline{P}_{max} = \frac{F_0^2}{2\gamma} = 10\text{ W}$ and $\omega_0 = 10^6$.

$\therefore$ Work done $W = 10 \times \frac{2\pi}{10^6} = 2\pi \times 10^{-5}$ W.

3(a) Since linear density $\rho = kx$, mass of the wire is a function of x.

Total mass $m = \int_0^L \rho\, dx = \int_0^L kx\, dx = \left.\frac{kx^2}{2}\right|_0^L = \frac{kL^2}{2}$

At a distance x from one end, the velocity is

$$v = \sqrt{\frac{T}{\rho}} = \sqrt{\frac{T}{kx}}$$

$$\frac{dx}{dt} = \sqrt{\frac{T}{kx}}$$

If τ denotes the time to travel distance L, we have

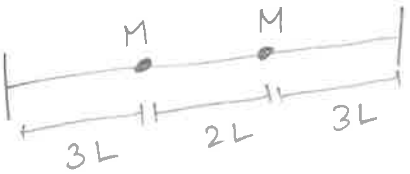
$$\sqrt{kx}\, dx = \sqrt{T}\, dt$$

Integrating both sides,

$$\int_0^L \sqrt{kx}\, dx = \int_0^\tau \sqrt{T}\, dt$$

Using $L = \sqrt{\frac{2m}{k}}$, we obtain $\tau = \frac{2}{3}\sqrt{\frac{2mL}{T}}$

(b)



To obtain eqns of motion, consider a particular configuration such as

Then $T \sin\theta_1 + T\sin\theta_2 = -M\frac{d^2y_1}{dt^2}$ and

$$T\sin\theta_1 - T\sin\theta_2 = -M\frac{d^2y_2}{dt^2}$$

We have, from the figure, $\sin\theta_1 \simeq \tan\theta_1 = \frac{y_1}{3L}$

$$\sin\theta_2 \simeq \tan\theta_2 = \frac{y_1 - y_2}{2L}$$

Then, the eqns. of motion are

$$T\left(\frac{y_1}{3L} + \frac{y_1 - y_2}{2L}\right) = -M\ddot{y}_1$$

$$T\left(\frac{y_2}{3L} - \frac{y_1 - y_2}{2L}\right) = -M\ddot{y}_2$$

For normal modes, assume solutions of the form

$y_1(t) = A\,e^{i\omega t}$ and $y_2(t) = B\,e^{i\omega t}$.

Substituting these in the eqns of motion and simplifying, we get

$$\underbrace{\begin{pmatrix} \frac{5T}{6L} - M\omega^2 & -\frac{T}{2L} \\ -\frac{T}{2L} & \frac{5T}{6L} - M\omega^2 \end{pmatrix}}_{\varphi} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Non-trivial solutions exist only if $|\varphi| = 0$.

This condition gives, $\omega_1^2 = \frac{4T}{3ML}$ and $\omega_2^2 = \frac{T}{3ML}$.

4(a) velocity of wave in medium 3 = $v_3 = 6\omega$

" " " " medium 1 = $v_1 = \omega/6$

No energy loss condition is $Z_2 = \sqrt{Z_1 Z_3}$, where $Z = \rho c$

$\therefore$ we have $\rho_2 v_2 = \sqrt{\rho^2 v_1 v_3} = \rho\omega$

$$\therefore \quad v_2 = \frac{\rho\omega}{\rho_2}$$

According to no-energy-loss condition, length of medium 2 is equal to $\lambda_2/4$.

$$\therefore \quad \frac{\lambda_2}{4} = \frac{2\pi\rho}{4\rho_2} = \frac{\pi}{2}\frac{\rho}{\rho_2}.$$

(b) See weblink given in problem sheet 6 (question 3).

5(a) μ = rigidity modulus = $\dfrac{F/A}{\theta}$

$\theta \rightarrow$ angle of shear.

For a cube of side L, $A = L^2$.

Substituting all the values, volume is $L^3 = \left(\frac{6}{\pi}\right)^{3/2} \times \frac{1}{1000}$

$L^3 \simeq 2.8 \times 10^{-3}\ m^3$.

(b) This problem was done in the class.

Refer to class notes or the web links given in problem sheet 7.