Indian Institute of Science Education and Research Pune
End-semester Exam, Jan (2019) semester.

Course name: Waves and Matter — **Course code:** PHY102
Date: 27.4.2019 (3:00 to 5:00 PM) — *Duration:* 2 hours
Instructor : M. S. Santhanam — Maximum marks: 60

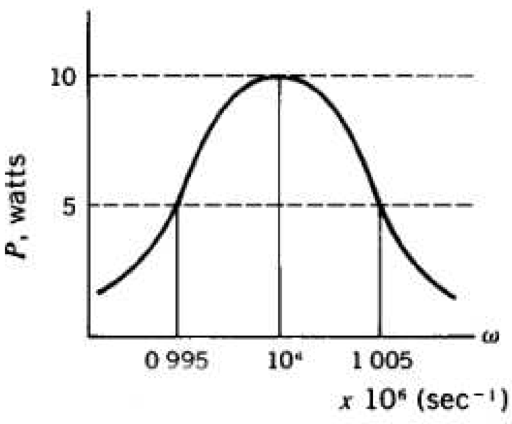
- This question paper has 5 questions. All of them are compulsory.
- If you draw sketches as an answer, label the axes. No marks without axes labels.
- SHOW ALL THE STEPS CLEARLY in your calculations while arriving at an answer.
- Unless specified otherwise, all the symbols have their usual meanings.
- Use the same symbols and notation given in the question. Do not use your own.

1.(a) Consider two oscillators in perpendicular directions. Their solutions are $x(t) = A\sin\omega t$ and $y(t) = -2A\sin\omega t$. Sketch the Lissajous figure for this situation.
(b) For the problem in (a), show by explicit calculation that the direction of motion (on the Lissajous figure) is anti-clockwise. Mark the direction.
(c) Sketch the velocity resonance curve for a damped and forced oscillator, for two cases with damping coefficients γ_1 and γ_2, such that $\gamma_2 > \gamma_1$.
(d) The amplitude of a damped oscillator is A at initial time. It decreases to $1/e$ of its initial value after N complete periodic cycles. If T_0 is the timeperiod without damping and T_d is the timeperiod with damping, show that

$$\frac{T_0}{T_d} = \sqrt{1 + \frac{1}{4\pi^2 N^2}}$$

(3+3+2+4)

2.(a) For a forced and damped oscillator, sketch the solution $x(t)$ for the case of critical damping.
(b) The phase velocity of ripples on water is given by $v_p = (kS/\rho)^{1/2}$, where S is the surface tension of water, ρ is its density and k is the wave number. Find the group velocity in terms of v_p.
(c) The graph shows the mean power absorbed by an oscillator when driven by a force of constant magnitude and variable angular frequency ω.
At exact resonance, how much work per cycle is being done against the resistive force ? What is the

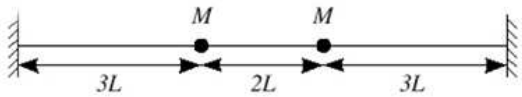


total mechanical energy of the oscillator ? (3+3+6)

3(a) A wire of length L and mass m has variable linear density $\rho = kx$ where k is a constant and x is the distance from one end of a string. Assuming that the wire is under tension T, show that the time taken for a pulse to travel from one end of wire to the other end is $(2/3)\sqrt{2mL/T}$.

(b) A wave passes from through three successive media without loss of energy. The three media characterised by impedence Z_1, Z_2 and Z_3. The waveform in the first medium is $y(x,t) = A\ \sin(9t - 6x)$ and in the third medium is $y(x,t) = A\ \sin(4t - x/6)$. If the density of the media ρ_1, ρ_2 and ρ_3 are such that $\rho_1 = \rho_3 = \rho$. Find the velocity of the wave in the second medium as a function of ρ_2.

(c) Two beads each of mass M are tied to a string of total length $8L$, as shown below. There is uniform tension T in the string. Find the frequencies of two normal modes. (3+3+6)



4(a) Find the Fourier series of the trapezoidal wave defined by the function,

$$\begin{aligned} f(x) &= x, \quad (0 \le x \le 1), \\ &= 1, \quad (1 \le x \le 2), \\ &= 3 - x, \quad (2 \le x \le 3). \end{aligned}$$

(b) A closed loop of uniform string is rotated rapidly at angular velocity ω. The mass of the string is m and the radius is R. A tension T is set up along the circumference in the string as result of its rotation. Show that the tension in the string is $m\omega^2 R/2\pi$. (6+6)

5(a) Consider a steel cube and a force of 8×10^6 N is applied tangentially (shear force) on one face of the cube. The angle of shear is about 0.3^o. Find the volume of the original cube. The rigidity modulus of steel is $80 \times 10^9\ N/m^2$. Compute an approximate (numerical) answer.

(b) A beam of length L is supported at its two extreme ends. Determine the deflection $y(x)$ and maximum deflection δ in this beam of length L carrying a uniformly distributed load W applied over its entire length. Specify the boundary conditions you apply to solve this problem. (4+8)