PH-3214: Problem Sheet

IISER, Pune. (April, 2025)

(NOTE : Some of these problems are taken from standard quantum mechanics books. Try problems from R. Shankar's book and from the book by Griffiths.)

1. Use WKB method to find the allowed energies of harmonic oscillator.

2. Use WKB method to find the allowed energies of power-law potential

$$V(x) = \alpha|x|^{\nu}, \text{ with } \nu > 0$$

Answer:

$$E_n = \alpha C[(n+1/2)\hbar]^{\frac{2\nu}{2+\nu}}$$

In this, C is an integral, which need not be evaluated.

3. A particle of mass m is in the ground state of $1D$ infinite well. At time $t = 0$, a brick is dropped in to the well so that

$$V(x) = \begin{cases} V_0, & \text{if } 0 \leqslant x \leqslant a/2 \\ 0, & \text{if } a/2 \leqslant x \leqslant a \\ \infty, & \text{otherwise.} \end{cases}$$

In this, $V_0 \ll E_1$. After time T, the brick is removed. Energy of the particle is measured. What is the probability that the energy is now E_2.

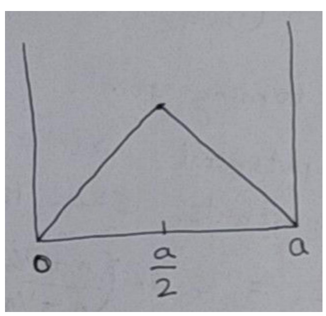
4. Use variational technique to find ground state energy for the Hamiltonian

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - \alpha\delta(x)$$

Use Gaussian trial wave function $\psi_t = Ae^{-bx^2}$.

5. Find upper bound on the ground state energy of 1D infinite square well using triangular wave function of the form

$$\begin{aligned} \psi_t &= Ax, \quad (0 \leqslant x \leqslant a/2) \\ &= A(a-x), \quad \left(\frac{a}{2} \leqslant x \leqslant a\right) \\ &= 0, \quad \text{otherwise.} \end{aligned}$$



6. Use variational technique and Gaussian trial wavefunction to obtain upper bound on ground state energy for
(a) $V(x) = \alpha|x|$,
(b) $V(x) = \alpha x^4$.

7. A particle in the harmonic oscillator potential

$$V(x) = \frac{1}{2}m\omega^2 x^2$$

starts in the ground state, $|0\rangle$. Starting at $t = 0$, a time-dependent perturbation is applied given by

$$H' = Ax^3 e^{-\gamma t}$$

Calculate the probability for the particle to make a transition to an excited state $|n\rangle$ after a long time. How do you define "long time" in this problem.

8. A hydrogen atom in the ground state is placed in a uniform electric field in the z-direction,

$$\varepsilon = \varepsilon_0 e^{-t/\tau}$$

This is turned on at $t = 0$. What is the probability that the atom is excited to the $2p$ state at $t \gg \tau$.

9. (Slightly difficult problem) For spherically symmetric potential, WKB, can be applied to radial part with $l = 0$. Use

$$\int_0^{r_0} p(r)dr = \left(n - \frac{1}{4}\right)\pi\hbar$$

where r_0 is the turning point.
Apply this to the potential $V(r) = V_0 \ln(r/a)$, where V_0 and a are constants. Show that

$$E_{n+1} - E_n = V_0 \ln\left(\frac{n + 3/4}{n - 1/4}\right).$$

10. Consider a system of two identical spin-1 particles. Find the spin states for this system that are symmetric or antisymmetric with respect to exchange of the two particles.
Sample Answer: Let's say, the first spin-1 particle can be in states $|1,-1\rangle_1, |1,0\rangle_1$ or $|1,1\rangle_1$. Similarly, the second spin-1 particle can be in states $|1,-1\rangle_2, |1,0\rangle_2$ or $|1,1\rangle_2$. There can be 9 possible product states. Now, example of a state that is symmetric with respect to exchange is
$|\psi_S\rangle = |1,1\rangle_1\ |1,-1\rangle_2 + |1,-1\rangle_1\ |1,1\rangle_2$. Similarly, construct other possible such states.

11. Consider two electrons moving in a central potential well in which there are only three single-particle states ψ_1, ψ_2 and ψ_3. (a) Write down all of the wave functions $\Psi(r_1, r_2)$ for the two-electron system.

12. Consider a system of three electrons described by three single-particle states ψ_1, ψ_2 and ψ_3. If they are indistinguishable particles, write down a general state if these are (a) bosons, (b) fermions.

13. Write down Hamiltonian for two non-interacting, identical particles in the infinite well. Construct two-particle states if they are (a) distinguishable, (b) identical bosons, (c) identical fermions. (See example 5.1 and problem 5.5 in Griffiths).

14. (slightly difficult problem) Consider two noninteracting particles of mass m in an infinite square well of width L. For the case with one particle in the single-particle state $|n\rangle$ and the other in the state $|k\rangle$ with $n \neq k$. Calculate the expectation value of the squared interparticle spacing $\langle (x_1 - x_2)^2 \rangle$, assuming (a) the particles are distinguishable (denoted by D), (b) the particles are identical spin-0 bosons, and (c) the particles are identical spin-1/2 fermions in a spin triplet state. Use bra-ket the notation as far as you can, but you will have to do some integrals.
Answer :
$\sqrt{\langle (x_1 - x_2)^2 \rangle_S} = 0.20L$
$\sqrt{\langle (x_1 - x_2)^2 \rangle_A} = 0.41L$
$\sqrt{\langle (x_1 - x_2)^2 \rangle_D} = 0.32L$