NAME : ROLL NUMBER :

PH-3214; Test : 2

IISER, Pune. (8 April, 2025) *Time: 50 minutes. Maximum Marks : 20.*

NOTE 1: Answer all the 5 questions. Use the same symbols/notation given in the questions.
NOTE 2: For questions 1 to 3, write the final answer ONLY inside the box. They will NOT carry any partial marks. Rough calculations can be done in separate sheet (but will not be evaluated).

1. A system represented by Hamiltonian operator $\widehat{H}$ changes abruptly (fast change) during the interval $t = -\epsilon/2$ to $t = \epsilon/2$, where $\epsilon \ll 1$. Let the states before and after the change be represented by $\psi(-\epsilon/2)$ and $\psi(\epsilon/2)$. Write down (do not derive) the relation between these states. (2)

$$\psi(\varepsilon/2) = \psi(-\varepsilon/2)$$

2. An harmonic oscillator with frequency ω is subjected to time dependent perturbation of the form $-\epsilon_0 X e^{-t^2/\tau^2}$. In this, ϵ_0 is a constant, and τ is the timescale of perturbation. Without any derivation, answer the following: (1+1+2)

(a) What is the time-period T of the system without the perturbation.

$$T = 2\pi/\omega$$

(b) Harmonic oscillator is in state $|m\rangle$ before the adiabatic perturbation begins. What is the final state of harmonic oscillator.

$$|m\rangle$$

(c) What is the condition on the timescales for which adiabatic theorem will apply.

$$2\pi/\omega << \tau$$

3. A system with time-dependence is given by $H(t) = H^0 + H^1 f(t)$, where $f(t)$ is a periodic perturbation with timescale of τ. Let the initial state be $|i_0\rangle$ with energy E_0. Due to selection rules, transitions are allowed only to states $|j_1\rangle, |j_2\rangle, |j_3\rangle$ with energies E_1, E_2, E_3. Answer the following questions. (1+2+2)

(a) For the system to make a transition to $|j_3\rangle$, what is the condition on energies E_0, E_1, E_2, E_3.

$$E_3 - E_0 = \hbar\omega$$

(b) After applying the perturbation $H^1 f(t)$ starting from $t = 0$, the probability $P_{i \to j_2}(t)$ for transition to state $|j_2\rangle$ is evaluated at times t_1 and t_2 such that $t_1 > t_2$. At what time (t_1 or t_2) is the probability $P_{i \to j_2}(t)$ highest ? How to increase this probability even closer to 1 ?

$P_{i \to j_2}$ highest at t_1. It gets close to 1 as $t \to \infty$.

(c) Let $f(t) = f_0$ (time-independent constant). Then, at very short times $t \ll 1$, what is the probability of initial state making a transition to state $|j_3\rangle$. Give an expression for this probability.

$$P_{i_0 \to j_3}(t) = |\langle i_0 | H^1 | j_3 \rangle|^2 f_0^2 t^2/\hbar^2$$

NOTE 3: Questions 4-5 require detailed answers with all the steps. Partial marks will be given for Questions 4 and 5, provided your calculations are in the right direction.

4. Consider a system subject to perturbation $H^1(t) = H^1\delta(t)$. If system is in state $|i\rangle$ for $t < 0$, then use first order perturbation theory to determin the probability to find the system in state $|f\rangle$ for $t > 0$. (5)

5. Show that the differential scattering cross-section $d\sigma(\theta)/d\Omega$ satisfies

$$\frac{d\sigma(0)}{d\Omega} \geq \frac{K^2\sigma^2}{16\pi^2},$$

where σ is the total cross-section. (5)

Start writing your answers for questions 4 and 5 in the space given below

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4. $H^1(t) = H^1\,\delta(t)$

Amplitude for transition from i^0 to f^0:

$$d_{f^0}(t) = -\frac{i}{\hbar}\int_{-t}^{t} \langle f^0 | H^1(t) | i^0\rangle\, e^{i\omega_{fi}t'}\, dt'$$

$$= -\frac{i}{\hbar}\int_{-t}^{t} \langle f^0 | H^1 | i^0\rangle\, \delta(t')\, e^{i\omega_{fi}t'}\, dt'$$

Due to $\delta(t)$, this integral is easy to perform. Just substitute $t'=0$. This gives,

$$d_{f^0}(t) = -\frac{i}{\hbar}\langle f^0 | H^1 | i^0\rangle$$

$$\therefore\ P_{i^0\to f^0}(t) = \frac{1}{\hbar^2}\left|\langle f^0 | H^1 | i^0\rangle\right|^2.$$