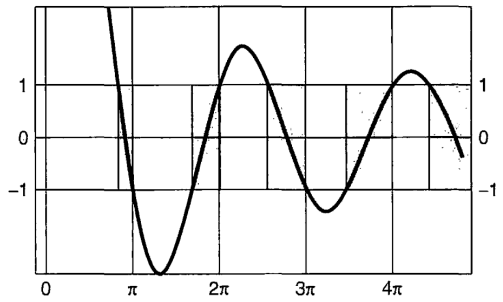
PH-3214; Test : 1

IISER, Pune. (4 February, 2025)

Time: 50 minutes. Maximum Marks : 25.

Answer all the questions. Show **all the steps** of your calculation.
Marks will be deducted if all steps not shown. For sketches, label the axes.

1. Examine the following figure that depicts the allowed and forbidden bands for a periodic potential with a series of delta functions.



This figure shows $f(z) = \cos z + 10(\frac{\sin z}{z})$ (on y-axis) as a function of $z = \sqrt{2mE}/\hbar$. In this m and E represent mass and energy of the particle. Give brief answers to the following questions :

(a) What is the physical meaning of allowed and forbidden bands.
(b) What happens to width (in energy) of forbidden bands in the limit when particle has large energy.
(c) Based on this figure, obtain an expression for the largest allowed energy in the first allowed band.
(2+2+2)

2. Let s_1 and s_2 represent two spins. Write down (need not derive) the product basis states (as column vectors) for the combined Hilbert space. (4)

3. Let S_1 and S_2 represent two spins such that $S = S_1 + S_2$. Obtain the eigenvalue for $S_z|++\rangle$. Show all the steps. (4)

4. Consider a spin-$\frac{1}{2}$ particle placed in an external magnetic field $\mathbf{B} = B_0\widehat{k}$. The Hamiltonian is given by $H = -\boldsymbol{\mu}.\mathbf{B}$, where $\boldsymbol{\mu}$ is the magnetic moment due to spin. Find the energy eigenvalues of this system. (5)

5. A coherent state can be expressed as

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle.$$

If α and β are two coherent states, find $|\langle\alpha|\beta\rangle|^2$. (6)