Indian Institute of Science Education and Research Pune
Mid-semester Exam, Jan (2025) semester.

Course name: Quantum Mechanics II | **Course code:** PH-3214
Date: 19.2.2025 (3:00 PM to 5:00 PM) | *Duration:* 2 hours
Instructor : M. S. Santhanam | Maximum marks: 60

- Among questions 4 to 7, answer <u>ANY THREE</u> of them. All other questions are compulsory.
- If you draw sketches as an answer, label the axes. No marks without axes labels.
- SHOW ALL THE STEPS CLEARLY in your calculations while arriving at an answer.
- Unless specified otherwise, all the symbols have their usual meanings.
- Use the same symbols and notation given in the question. Do not use your own.

For states involving sum of angular momenta, use ONLY one of the following notation depending on the problem and requirement :

$$|l\ m,\ l_1\ l_2\rangle, \qquad |lm\rangle, \qquad |l_1\ m_1,\ l_2\ m_2\rangle.$$

l could be s as well in some cases. DO NOT use any other notation.

1. What are the possible values of l and m for an atom consisting of three p electrons and one f electron. What is the dimension of associated Hilbert space. (3+3)

2. Two p electrons are in the coupled angular momentum state $|l\ m,\ l_1\ l_2\rangle = |2,\ 1;\ 1,\ 1\rangle$. What is the joint probability of finding the two electrons with $L_{1z} = \hbar$ and $L_{2z} = 0\hbar$. (6)

3. Show that $\langle\alpha|x|\alpha\rangle = \sqrt{\frac{2\hbar}{m\omega}}\ \mathrm{Re}(\alpha)$, where α is the coherent state of harmonic oscillator. (6)

4. Let the Hamiltonian for an electron in an external magnetic field be $H = -\boldsymbol{\mu}.\mathbf{B}$, where $\boldsymbol{\mu}$ is the magnetic moment. Let $\boldsymbol{B} = B_0\widehat{\boldsymbol{k}}$ be the external magnetic field, and the initial state of electron is

$$\psi(t=0) = \frac{i}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}.$$

(a) Determine the electron's time period of precession T.
(b) Show that $\psi(t = 3T/4) = A|\downarrow\rangle_y$, where $|\downarrow\rangle_y$ is the eigenvector of σ_y Pauli matrix.
(c) Show that $A = e^{-i3\pi/4}$. (1+4+2)

5. Hamiltonian of a system consisting of two spin-$\frac{1}{2}$ particles is given by

$$H = A\ \mathbf{S_2}.\mathbf{S_1} + B\left(S_{1z} + S_{2z}\right),$$

where A, B are constants.
(a) Express $\mathbf{S_2}.\mathbf{S_1}$ in terms of total spin S.
(b) What are total-s basis states for this problem.
(c) Determine all the eigenvalues. (1+2+4)

6. Consider the perturbed infinite well system given by:

$$\begin{aligned} H &= \frac{p^2}{2m} + \frac{kx}{L}, \qquad && (0 < x < L), \\ &= \infty, && (x \leq 0, \quad \text{and} \quad x \geq L). \end{aligned}$$

Determine the first order estimate for energy using perturbation theory. (7)

7. For the linear potential given by $V(x) = \alpha|x|$, where α is a parameter, estimate the ground state energy using variational method. Use the trial wave function $\psi_{trial} = A\ e^{-bx^2}$. (7)

8. Consider a system of particles that obey $\frac{1}{2} \otimes 1 = \frac{3}{2} \oplus \frac{1}{2}$.
(a) Determine the dimension of Hilbert space of this problem.
(b) List all the product basis states and coupled basis states.
(c) Determine all the CG coefficients. (1+4+6)

Some useful information. You can use this information in your calculation (if neeeded):

1) $J_{\pm}|j, m\rangle = \hbar\sqrt{(j \mp m)(j \pm m + 1)}\ |j, m \pm 1\rangle$

2) For an infinite well problem. Potential is zero in $0 < x < a$. The eigenvalues are :

$$E_n = \frac{n^2 h^2}{8ma^2}.$$

3) Symmetry relation for CG coefficients :

$$\langle j_1 m_1, j_2 m_2 | j m\rangle = (-1)^{j_1 + j_2 - j}\ \langle j_1(-m_1), j_2(-m_2)| j(-m)\rangle$$