Indian Institute of Science Education and Research Pune

End-semester Exam, Jan (2025) semester.

Course name: Quantum Mechanics II | **Course code:** PH-3214

Date: 22.4.2025 (3:00 PM to 5:00 PM) | *Duration:* 2 hours

Instructor : M. S. Santhanam | Maximum marks: 50

- Among questions 1 to 6, answer ANY FIVE of them. Questions 7 to 9 are compulsory.
- Use the same symbols and notation given in the question. Marks will be cut for using different symbols/notations.
- SHOW ALL THE STEPS CLEARLY in your calculations while arriving at an answer.
- If you draw sketches as an answer, label the axes. No marks without axes labels.
- Unless specified otherwise, all the symbols have their usual meanings.

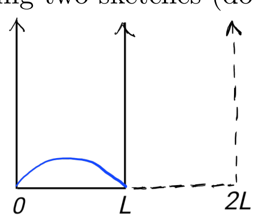
1. Let $s_1 = 1$ and $s_2 = \frac{1}{2}$ be a two-spin system. List all its product states and coupled states. (5)

2. (a) Sketch the potential $V(x) = \alpha x^4$, for $\alpha > 0$. (b) For a particle of energy E, write an expression for classical turning points. (c) Mark the turning points on the x-axis of the graph drawn for part (a) of this questtion. (3+2)

3. A 1D quantum system is in state $|n\rangle$ with energy E_n. A time-independent perturbation $H^1 = V_0 x^2$ is applied, where V_0 is the strength of perturbation. Calculate first-order energy correction, and write down the energy of the state $|n\rangle$ after perturbation is applied. Under what condition is this result valid ? (3+2)

4. Consider 3 identical fermions represented by orthogonal states $\psi_{n_1}(x), \psi_{n_2}(x)$ and $\psi_{n_3}(x)$. Write an expression for a fermionic state $\Psi(x_1, x_2, x_3)$ that accounts for its symmetry property. What is the normalisation constant of this state. (3+2)

5. Consider the infinite well between $x = 0$ and $x = L$ with ground state sketched in it (figure shown below). The left side wall at $x = 0$ is unchanged. The right wall at $x = L$ is moved to $x = 2L$. Then, answer the following questions by drawing two sketches (do not derive anything);



(a) The wall is moved *slowly* to $x = 2L$. First sketch the modified infinite well, and on top of it sketch the new ground state at the end of the process.
(b) The wall is moved *very fast* to $x = 2L$. First sketch the modified well, and on top of it sketch the new ground state *immediately* after the wall is moved to $x = 2L$. (5)

6. Let $H|n\rangle = E_n|n\rangle$, with $n = 0, 1, 2, \ldots$. Starting from $\langle\psi|H|\psi\rangle$, where ψ is some trial state, show that upper bound to ground state energy E_0 is (5)

$$E_0 \leq \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle}.$$

7. Consider the Hamiltonian for a linear potential given by

$$H = \frac{p^2}{2m} + \alpha|x|,$$

where $\alpha > 0$ is a parameter. Use WKB method to estimate the energy of this system. (Do not derive WKB formulas). (7)

8. Consider a bosonic system of two identical particles in an infinite square well of width L. One particle is in ground state $\phi_1(x)$ and the other is in an excited state $\phi_2(x)$.
(a) Assuming the positions of particles to be x_1 and x_2, write down the normalised symmetric state $\Psi(x_1, x_2)$ for this system.
(b) Now, let $x_1 = x_2 = x$. Show that the probability density for the two particles to be in the same position x is of the form,

$$P_S = C\ \sin^2(\theta)\ \sin^2(2\theta),$$

and explicitly obtain the values of C and θ in terms of L and x.
(c) Evaluate this probability at $x = L/3$. (Final answer should be a number. Calculator not required.) (3+3+2)

9. A particle in the harmonic oscillator potential $V(x) = \frac{1}{2}m\omega^2x^2$ is in its ground state $|0\rangle$. At $t = 0$, a time-dependent perturbation $H^1 = Ax^3e^{-\gamma t}$ is applied. In the limit $t \to \infty$, show that the transition probability from ground state $|0\rangle$ to $|n\rangle$ is

$$P_{0\to n} = \frac{3A^2\hbar}{8m^3\omega^3}\left(\frac{1}{n^2\omega^2 + \gamma^2}\right)(2\delta_{n,3} + 3\delta_{n,1}).$$

(10)

Some useful information. You can use this information in your calculation (if neeeded):

- Eigenstate of infinte well system of width L: $\phi_n(x) = \sqrt{\frac{2}{L}}\ \sin\left(\frac{n\pi x}{L}\right)$

- If $|n\rangle$ and $|0\rangle$ are harmonic oscillator states, here is a useful matrix element :

$$\langle n|x^3|0\rangle = \left(\frac{\hbar}{2m\omega}\right)^{3/2}\langle n|a^\dagger a^\dagger a^\dagger + a^\dagger a a^\dagger + a a^\dagger a^\dagger|0\rangle$$

- Creation and annihilation operators : $a|n\rangle = \sqrt{n}|n-1\rangle$, $\quad a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$.

- A useful integral :

$$\int_0^1 \sqrt{1-y}\ dy = \frac{2}{3}$$