PHY3124; Test : 2 *IISER, Pune. (7 November, 2024)*

Time: 50 minutes. Marks : 25.

Answer all the questions. Show all the steps of your calculation. Marks will be deducted for if all steps not shown. For sketches, label the axes.

1. If \mathbb{P} is the parity operator, what are its possible eigenvalues ? Determine the parity of the state $\psi(x) = e^{-x^2/2} \cos x$. (3)

2. A two dimensional harmonic oscillator has energy given by E = k + |m| + 1/2, where k = 0, 1, 2, ...,and m is the magnetic or azimuthal quantum number. For any given n = k + |m|, determine the degree of degeneracy ? (4)

3. Let $H \ \psi(x) = E \ \psi(x)$. Show that if parity operator \mathbb{P} commutes with H, then $\psi(-x)$ is also an eigenstate of H with same eigenvalue. (6)

4. Prove the delta function identity (a > 0): $\delta(x^2 - a^2) = \frac{1}{2a} [\delta(x - a) + \delta(x + a)]$ (6)

5. Let $U[R(\epsilon, \mathbf{k})]$ be the quantum operator corresponding to infinitesimal rotation by an angle ϵ about z-axis. Starting from $U^{\dagger}[R] P_y U[R]$, derive the commutator $[P_y, L_z] = i\hbar P_x$. (6)