PHY3124; Test : 1

IISER, Pune. (5 Sept, 2024)

Time: 50 minutes. Marks : 25.

Answer all the questions. Show all the steps of your calculation.
Marks will be deducted for if all steps not shown. For sketches, label the axes.

1. (a) What is the dimension of $\hbar$? (2)

2. Write the following in Dirac notation (assume ϕ and ψ to be function of x) :
(a) $\int_{-\infty}^{\infty} \widehat{O}_1^{\dagger}\phi^* \ \psi \ dx$

(b) $\int_{-\infty}^{\infty} \ \phi^* \ \phi \ dx$ (2+2)

3. Sketch the first three eigenstates $|u_n(x)|^2$, $(-L/2 \leq x \leq L/2)$, for quantum numbers $n = 1, 2, 3$ for the infinite square well potential. (3)

4. Show that the commutator $[\widehat{P}^2, \widehat{X}] = -2i\hbar\widehat{P}$. (5)

5. Let an eigenstate be $\psi(x) = A \ e^{ikx} \ e^{-\alpha x}$, where $\alpha > 0$ and $A > 0$ are constants. Calculate the probability density current associated with this state. (5)

6. Consider the standard infinite well system with potential $V(x) = 0$ for $-L/2 \leq x \leq L/2$. The system is in a state given by,

$$\psi(x) = \frac{1}{\sqrt{L}}.$$

Find an expression for the probability that $\psi(x)$ is in the ground state of infinite well system. The ground state of infinite well system is : $\phi_1(x) = \sqrt{1/L} \ \cos(\pi x/L)$ (6)