PH3124 : Assignment 2

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(NOTE : *This is a sample selection of problems. You must try out more problems from other text books as well. Books by R. Shankar, Walter Greiner and Griffiths are a good source of problems.*)

1. Show that invariance under time translation leads to energy conservation, i.e, $\langle \dot{H} \rangle = 0$.

2. If a Hamiltonian H is parity invariant, show that $\exp(-iHt/\hbar)$ is also parity invariant.

3. In (x, y, z) coordinate system, angular momentum operator is given by $L_z = XP_y - YP_x$. Perform a coordinate transformation to spherical polar coordinate system and show that $L_z \to -i\hbar \frac{\partial}{\partial \phi}$.

4. Starting from $U[R(-\epsilon_z \mathbf{k})]T(-\epsilon)U[R(\epsilon_z \mathbf{k})]T(\epsilon)$, show that $[P_y, Lz] = i\hbar P_z$ and $[P_x, Lz] = -i\hbar P_y$. In this, U and T are rotation and translation operators.

5. Do the problems 12.3.3 and 12.3.4 given in R. Shankar's book.

6. Solve the eigenvalue problem of a particle constrained to move on a circle of radius a.

7. A series of classical rotations leads to the following operation;

$$R(-\epsilon_y \mathbf{j})R(-\epsilon_x \mathbf{i})R(\epsilon_y \mathbf{j})R(-\epsilon_x \mathbf{i}) = R(-\epsilon_x \epsilon_y \mathbf{k})$$

From this, deduce that $[L_i, L_j] = i\hbar \sum_{k=1}^{3} \epsilon_{123} L_k$, where $i, j = 1, 2, 3$.

8. Do problems 12.5.2 and 12.5.3 from R. Shankar's book.

9. If $H = p^2/2m + V(r)$, then show that the Hamiltonian commutes with all three components of angular momentum operator.

10. Mathematically define space translation invariance. Show that translation invariance implies that mean momentum is conserved.

11. Translation operator (to order ϵ) is $T(\epsilon) = I - \frac{i\epsilon}{\hbar} G$. Use $T^\dagger(\epsilon)\, T(\epsilon) = I$ to show that $G^\dagger = G$.

12. Prove that if $[P, H] = 0$, and a system starts out in even/odd parity, then the system maintains its parity under time evolution.

13. Show the following delta function identities :

$$\delta(cx) = \frac{1}{|c|}\delta(x) \tag{1}$$

$$
\begin{aligned}
y\delta'(y) &= -\delta(y) \qquad (2) \\
y\delta(y) &= 0 \qquad (3)
\end{aligned}
$$

14. Solve the spectra of bound states for Dirac-delta function potential.

15. Consider $x = \rho \cos \phi$ and $y = \rho \sin \phi$. Show that the area element $dxdy$ transforms as $\rho d\rho d\phi$.