PH3124: Assignment-1

IISER, Pune. (August, 2024)

(NOTE : These are practice problems for you try out. This is not meant for evaluation. So, this need not be submitted.)

0. Try problems from (a) Book by R. Shankar, (b) Book by Griffiths. Stick to specific topics that was taught in class. Here are some sample problems given below to try out.

1. The classical Hamiltonian for the harmonic oscillator is,

$$H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2}.$$

Obtain the Hamiltonian operator in momentum representation. Derive every step of the calculation.

2. Starting from $\hat{P}|p\rangle = p|p\rangle$, show that $\langle x|p\rangle = \exp(ixp/\hbar)$.

3. A particle in the infinite square well has the wave function

$$\psi(x) = A\ x(a-x),$$

where A is the normalisation constant and $0 \leq x \leq a$. Find the probability for the system to be in the ground state and first excited state of the infinite well.

4. Show that in one dimensional problems with the Hamiltonian of the form $\hat{P}^2 + V(x)$, there is no degeneracy in the spectrum.

5. Show that for any normalised $|\psi\rangle$, $\langle\psi|H|\psi\rangle \geq E_0$, where E_0 is the lowest energy eigenvalue.

6. Calculate the normalised classical probability to find the particle in the region x and $x + dx$ in the infinite well potential.

7. Find the energy eigenvalue and the orthogonal eigenstates for $\hat{H} = \hat{p}^2/2m$.

8. Calculate the commutator $[\hat{x}, \hat{p}]$ and the anti-commutator defined as $[\hat{x}, \hat{p}]_+ = \hat{x}\hat{p} + \hat{p}\hat{x}$.

9. Show that the position operator in momentum representation is Hermitian ? Show that kinetic energy operator is Hermitian ?

10. Show that for Hermitian operators, the eigenvectors belonging to distinct eigenvalues are orthogonal.

11. If $\hat{X}|x\rangle = x|x\rangle$, show that for any potential operator of the form $V(x)$, $V(\hat{X})|x\rangle = V(x)|x\rangle$.

12. An electron is accelerated through a constant potential V. What is its De Broglie wavelength ? What is the probability current associated with this electron ?

13. Associated with a particle is an infinite plane wave of form

$$\psi(x,t) = A \ \exp\left(i(kx - \omega t)\right).$$

Show that phase velocity of matter waves is larger than the velocity of light (in vacuum).

14. What are the dimensions of $\hbar$ and energy E ?

15. Why do we require low intensity levels in a double slit experiment to infer that particles can behave as waves ?