Lecture 1.

Rejish Nath

Course Contents

1. Nature of light (waves and particles)
2. Maxwells equations and wave equation
3. Poynting vector
4. Polarization of light
5. Law of reflection and snell's law
6. Total Internal Reflection and Evanescent waves
7. Concept of coherence and interference
8. Young's double slit experiment
9. Single slit, N-slit Diffraction
10. Grating, Birefringence, Retardation plates
11. Fermat's Principle
12. Optical instruments
13. Human Eye
14. Spontaneous and stimulated emission
15. Concept of Laser

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Course Coordinator: Dr. Umakant Rapol

End-Sem Examination-30%
Mid-Sem Examination-30%
Quiz 1 - 20%
Quiz 2 - 20%

Contents

Literature:

1. Optics, (Eugene Hecht and A. R. Ganesan)
2. Optical Physics, (A. Lipson, S. G. Lipson and H. Lipson)
3. Optics, (A. Ghatak)

Introduction

Optics is all about light, its properties, the phenomena associated to it and the instruments to study those!

And God said, Let there be light: and there was light.

What is light?

Two perspectives in the 17th century:

- Corpuscular theory by Newton
- Wave theory by Huygens

- The principle of interference came later (Thomas Young, and Augustin Fresnel).

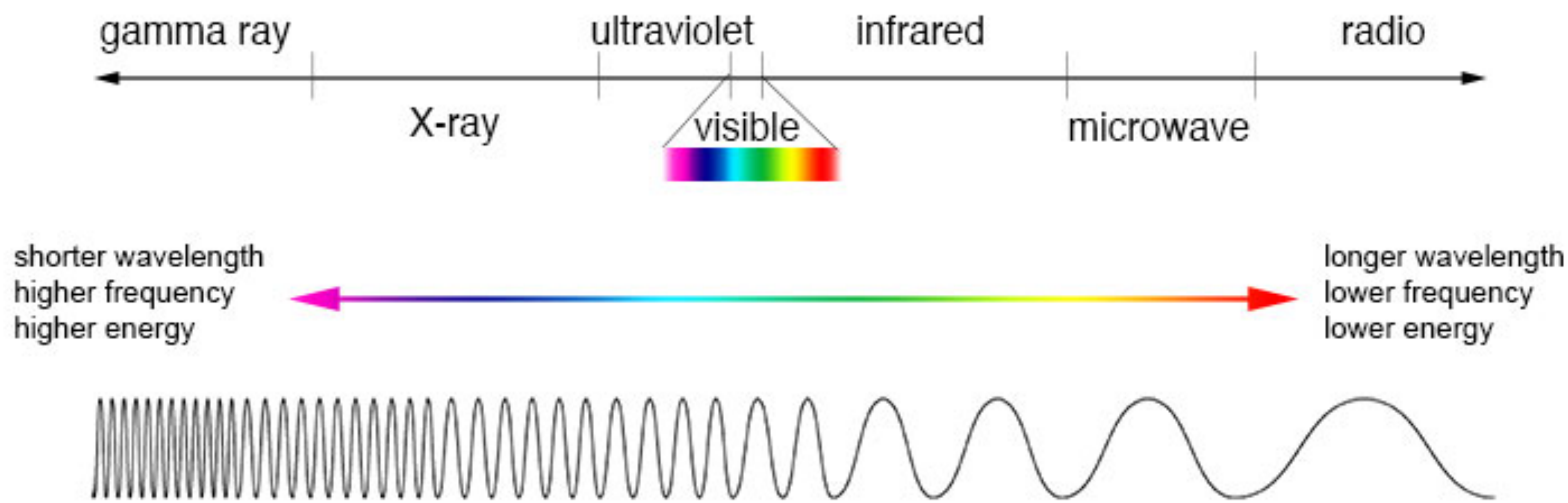
- Then comes the polarisation of light (Malu’s law experiment).

- Light as transverse waves has been brought up.

- Studies on speed of light, and seen it as an electro magnetic wave.

Wave nature of light

- Light is an electro magnetic wave.
- Electro-magnetic spectrum



- Maxwell's theory of light propagation and quantum theory of light (QED) can explain most of the phenomena associated with the light.

- Quantum electro-dynamics (QED) deals with how light interacts with matter.

Waves

- Longitudinal Waves:
  vibration of the medium is along the direction of propagation.
  e.g. Sound waves
- Transversal Waves:
  vibration of the medium is perpendicular to the direction of propagation.
  e.g. waves on a string, electro magnetic waves.
- Crucially note that it is not the medium that propagates, but the disturbance advances.In the case of light it is the magnitude of electric and magnetic fields, identical to the vertical displacement in a string.

Longitudinal Wave vs. Transverse Wave

Since the disturbance is moving,
it is a function of both space and time.

$$f(x,t)$$

gives the profile of the wave at any instant of time (wave function).

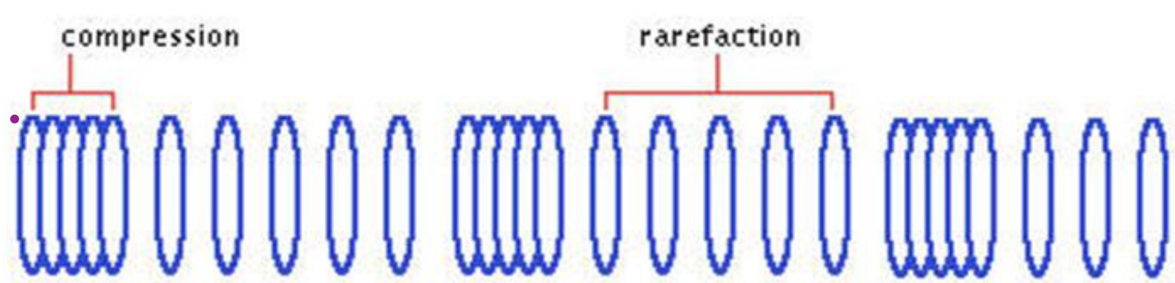


Figure 1: Longitudinal wave

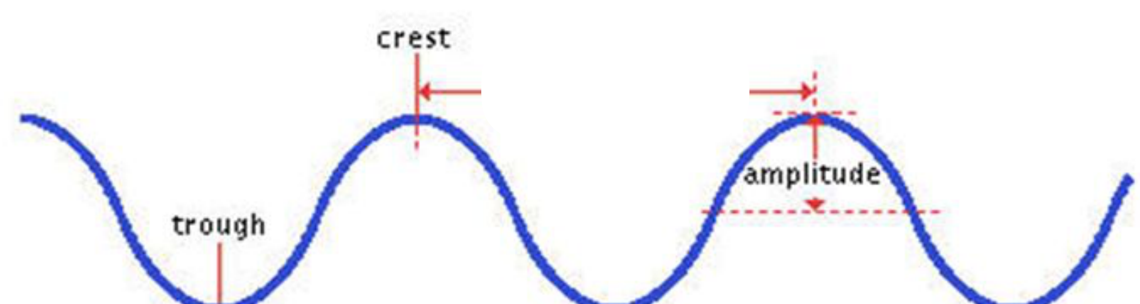


Figure 2: Transverse wave

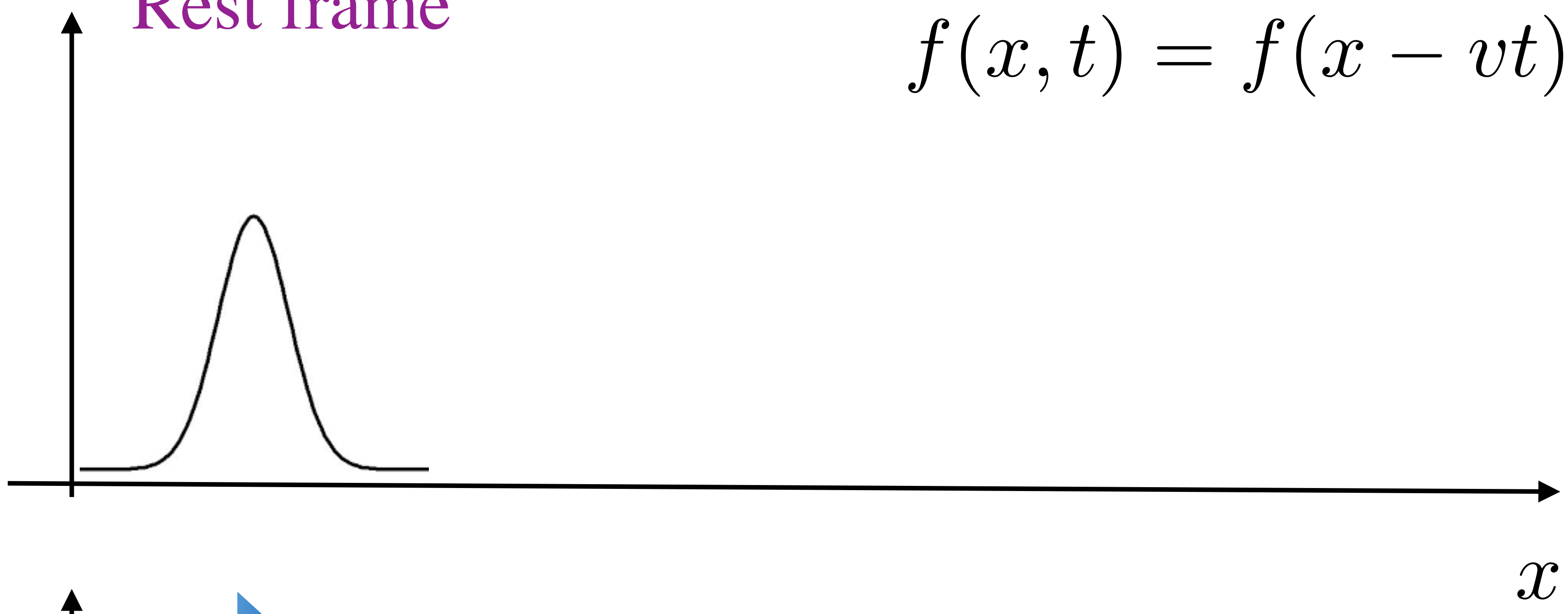
Waves

$f(x,t)$ We are talking about the travelling (or moving) waves. Let its velocity
assume that the shape of the wave is not changing.

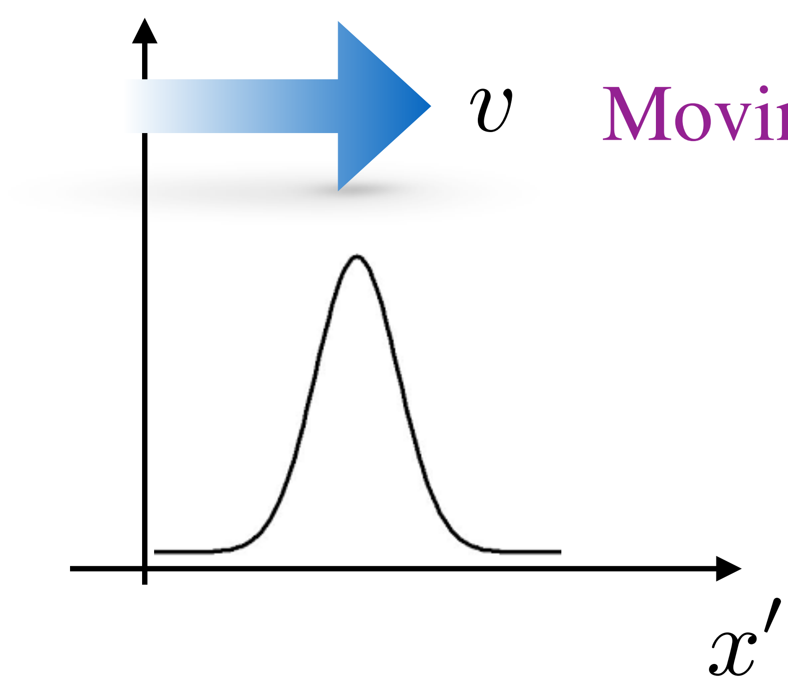
The distance covered in time *t* is *vt*.

$$f(x,t) = f(x - vt)$$

Rest frame



Moving frame



$$f(x') = f(x,0)$$

(independent of time)

Waves

Once you choose a wave shape: $f(x,0)$, the time propagation is just $f(x-vt)$.

Example: A Gaussian

$$f(x,t=0) = e^{-ax^2}$$

$$f(x,t) = e^{-a(x-vt)^2}$$

A gaussian disturbance or pulse propagating with a speed v.

Exercise: Verify by plotting the above function at different instant of times, that it is propagating!

What would $f(x+vt)$ do?

The wave (differential) equation

Aim: Can we get the wave function as a solution of a differential equation. **Then we can say** the differential equation describes the system at any instant of time.

Step 1.

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial (x \pm vt)} \times \frac{\partial (x \pm vt)}{\partial t} \qquad = \pm v \frac{\partial f}{\partial x}$$

Step 2.

$$\frac{\partial^2 f}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial x^2}$$

$$\boxed{\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}}$$

Wave equation

A homogeneous second order differential equation.

Harmonic waves

- The shape of the waves is of **sine** or **cosine** forms called the *sinusoidal* or simple *harmonic* waves.

- Any other wave shape can be constructed by mixing harmonic waves.

$$f(x,0) = A\sin(kx)$$

Since the argument is dimensionless, we are forced to multiply by a constant *k*, which has a unit of inverse length

amplitude of the sine wave.

Propagating wave:

$$f(x,t) = A\sin\left[k(x - vt)\right]$$

periodic in both space and time.

- The spatial period is called the wavelength "λ".

$$f(x,t) = f(x \pm \lambda, t) \qquad k\lambda = 2\pi$$

- The time period or temporal period "τ"

$$f(x,t) = f(x, t \pm \tau) \qquad kv\tau = 2\pi$$

Harmonic waves

$$k\lambda = 2\pi \qquad\qquad kv\tau = 2\pi$$

$$v\tau = \lambda$$

Introducing the temporal frequency $\nu = 1/\tau$

$$\boxed{v = \lambda\nu}$$

$$\omega = 2\pi\nu$$

Angular temporal frequency: $\omega = 2\pi/\tau$

$$\boxed{f(x,t) = A\sin(kx - \omega t)}$$

- Harmonic waves are infinitely extended.
- Single frequency waves are called *mono-chromatic* or *mono-energetic*.
- Practically we have only quasi-monochromatic sources, where there is a small band width of frequencies. (related to quantum mechanics)

Phase of the Harmonic wave

Lets take a harmonic wave:

$$f(x,t) = A\sin(kx - \omega t)$$

The argument,

$$\boxed{\phi(x,t) = kx - \omega t}$$

$$\phi(x,t) \in [0, 2\pi]$$

is called the phase of the wave.

Lets take another wave:

$$g(x,t) = A\cos(kx - \omega t)$$

What is the difference between these two waves?

A phase difference of $\pi/2$.

Phase of the Harmonic wave

$$\phi = kx - \omega t$$

the phase is varying with time, and the rate of change of phase with time at a fixed location.

$$\left|\frac{\partial \phi}{\partial t}\right|_x = \omega$$ the angular frequency

the rate of change of phase with distance keeping the time constant.

$$\left|\frac{\partial \phi}{\partial x}\right|_t = k$$ the wave number

Phase velocity: the velocity at which the point of *constant phase* propagates

$$v_p = \left(\frac{\partial x}{\partial t}\right)_\phi = \frac{\omega}{k}$$

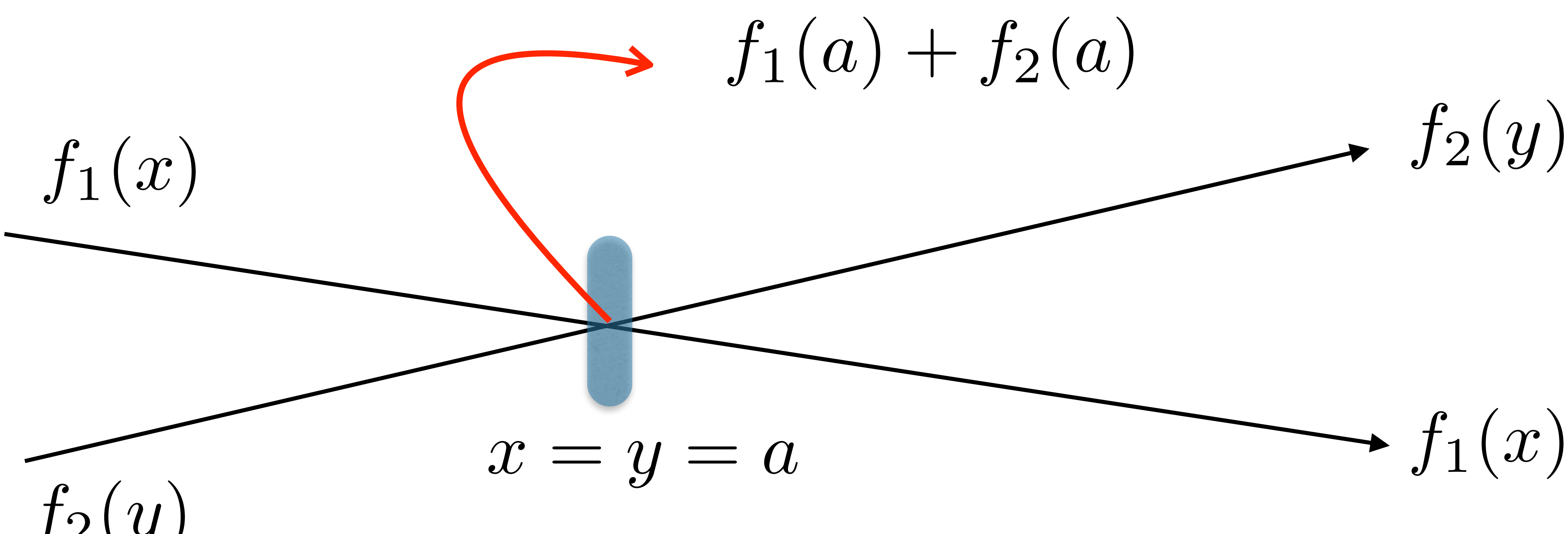
The superposition principle

◆ If $f_1(x)$ and $f_2(x)$ are two different solutions of the wave equation:

$$\boxed{\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}}$$

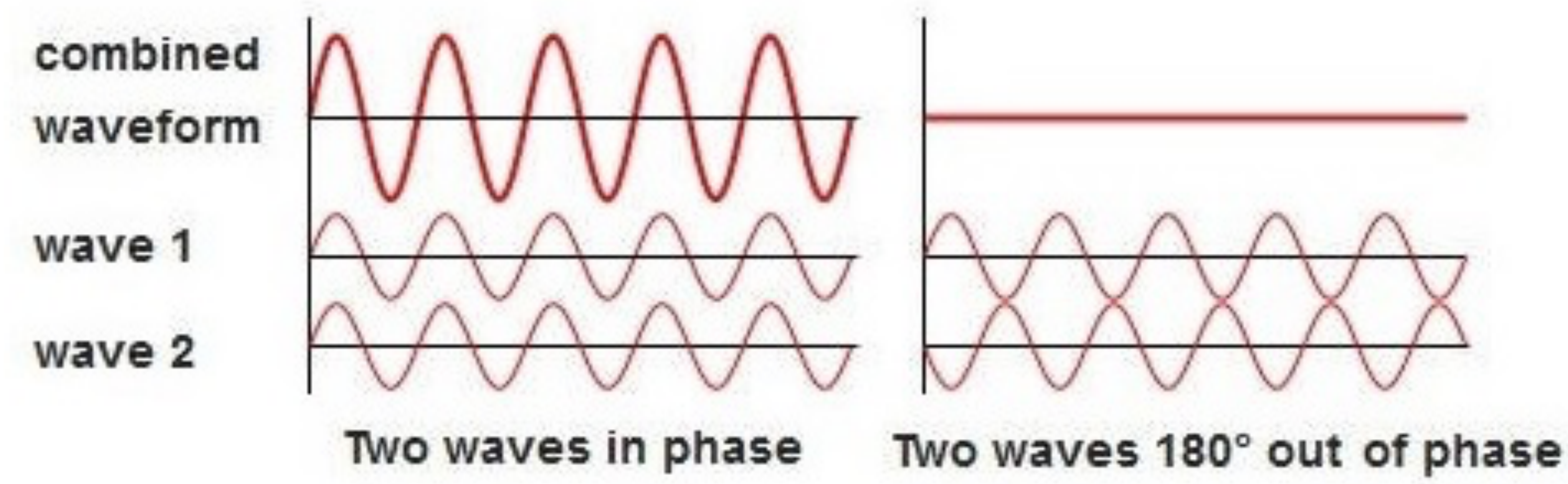
then $f_1(x) + f_2(x)$ is also a solution.

That means, the region of space in which the waves intersect, the amplitudes are added up.



The superposition principle

- The amplitudes are added up in the intersecting region.
- At the intersecting region, if the result is **maximum**, then the waves are **in phase (phase difference is zero)** at that point.
- If the result is **minimum and is zero**, then the waves are **out of phase by 180 degrees** at that point.



Complex representation of waves

$$e^{i\phi} = \cos\phi + i\sin\phi$$

$$\cos\phi = \text{Real}[e^{i\phi}]$$

$$\sin\phi = \text{Imag}[e^{i\phi}]$$

- Lets take the Harmonic wave:

$$f(x,t) = A\cos(kx - \omega t) = A \times \text{Real}\left[e^{i(kx-\omega t)}\right]$$

Typically we just avoid writing "Real […]", and simply write as

$$f(x,t) = Ae^{i(kx-\omega t)}$$

The *actual wave* is only then the real part of it.

Plane Waves (3D)

Let's consider a three dimensional solution of the form:

$$f(\mathbf{r}, t=0) = Ae^{i\mathbf{k}\cdot\mathbf{r}}$$

$$\mathbf{k}\cdot\mathbf{r} = xk_x + yk_y + zk_z$$

This is not same as

$$Ae^{ikr}$$

The phase of the wave is $\phi = \mathbf{k}\cdot\mathbf{r}$

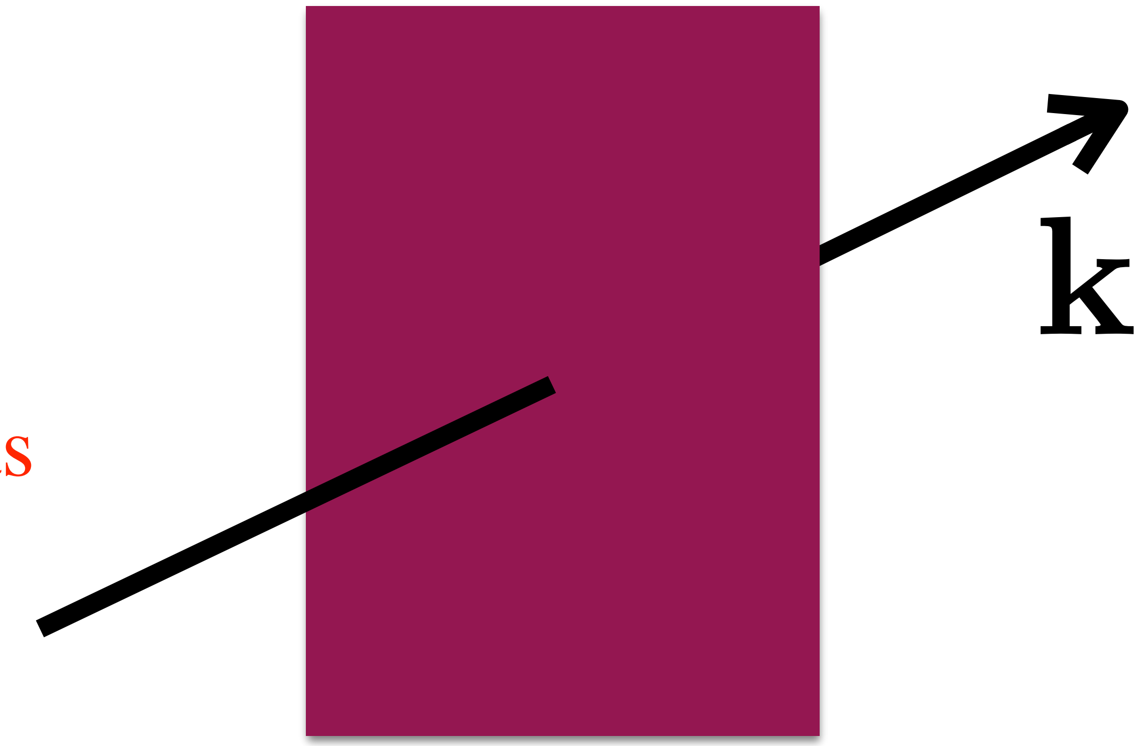
To simplify let's take
the wave-vector (propagation vector) as:

$$\mathbf{k} = k\ \hat{z}$$

The condition of constant phase provides as

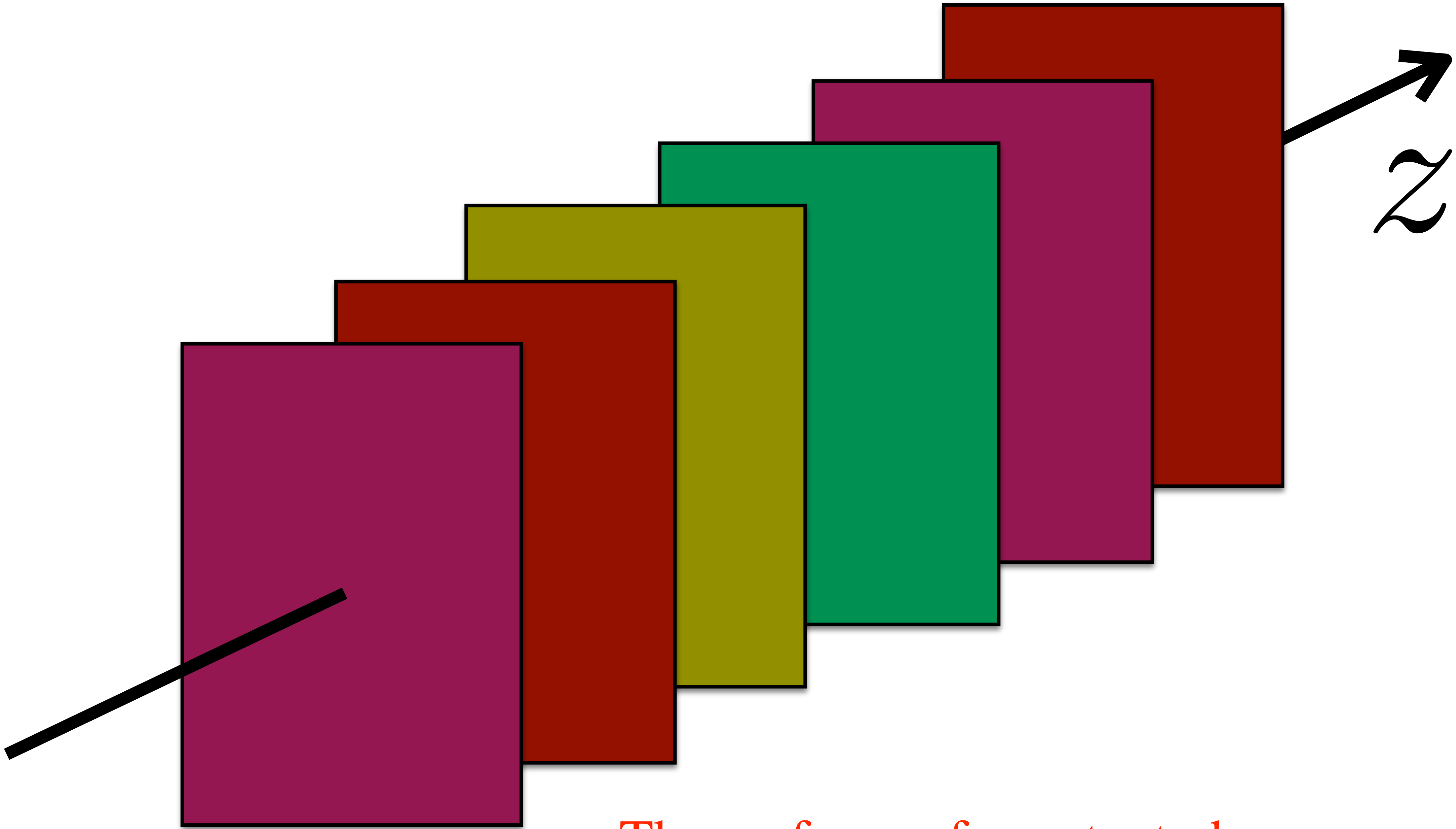
$$\boxed{\mathbf{k}\cdot\mathbf{r} = k \times z = \text{constant}}$$

defines **a plane.**



Plane Waves (3D)

- Each of these planes represent a constant phase.
- There are as many as infinity and infinitely extended in the x and y directions.



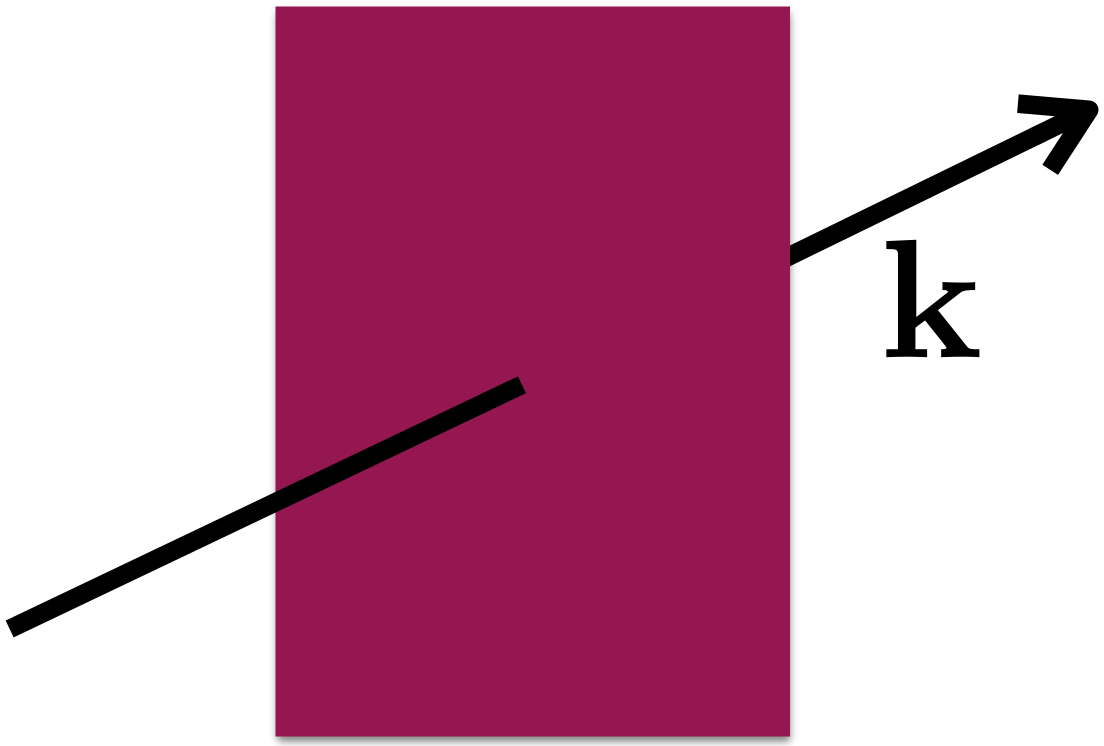
The surfaces of constant phase are called wave fronts.

Plane Waves (3D)

The equation of plane for an arbitrary wave vector:

$$\mathbf{k} \cdot \mathbf{r} = \text{constant}$$

$$\boxed{f(\mathbf{r}, t = 0) = A e^{i\mathbf{k}\cdot\mathbf{r}}}$$



Note that the wave function also has a constant value in this plane, if the amplitude A is a constant.

The time dependent version of the plane wave is

$$f(\mathbf{r}, t) = A e^{i(\mathbf{k}\cdot\mathbf{r} \pm \omega t)}$$

the direction of propagation is now given by the wave vector $\mathbf{k}$.

3D Wave Equation

$$\boxed{\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}}$$

Wave equation

A homogeneous second order differential equation.

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

(Laplacian operator)

$$\nabla^2 f = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

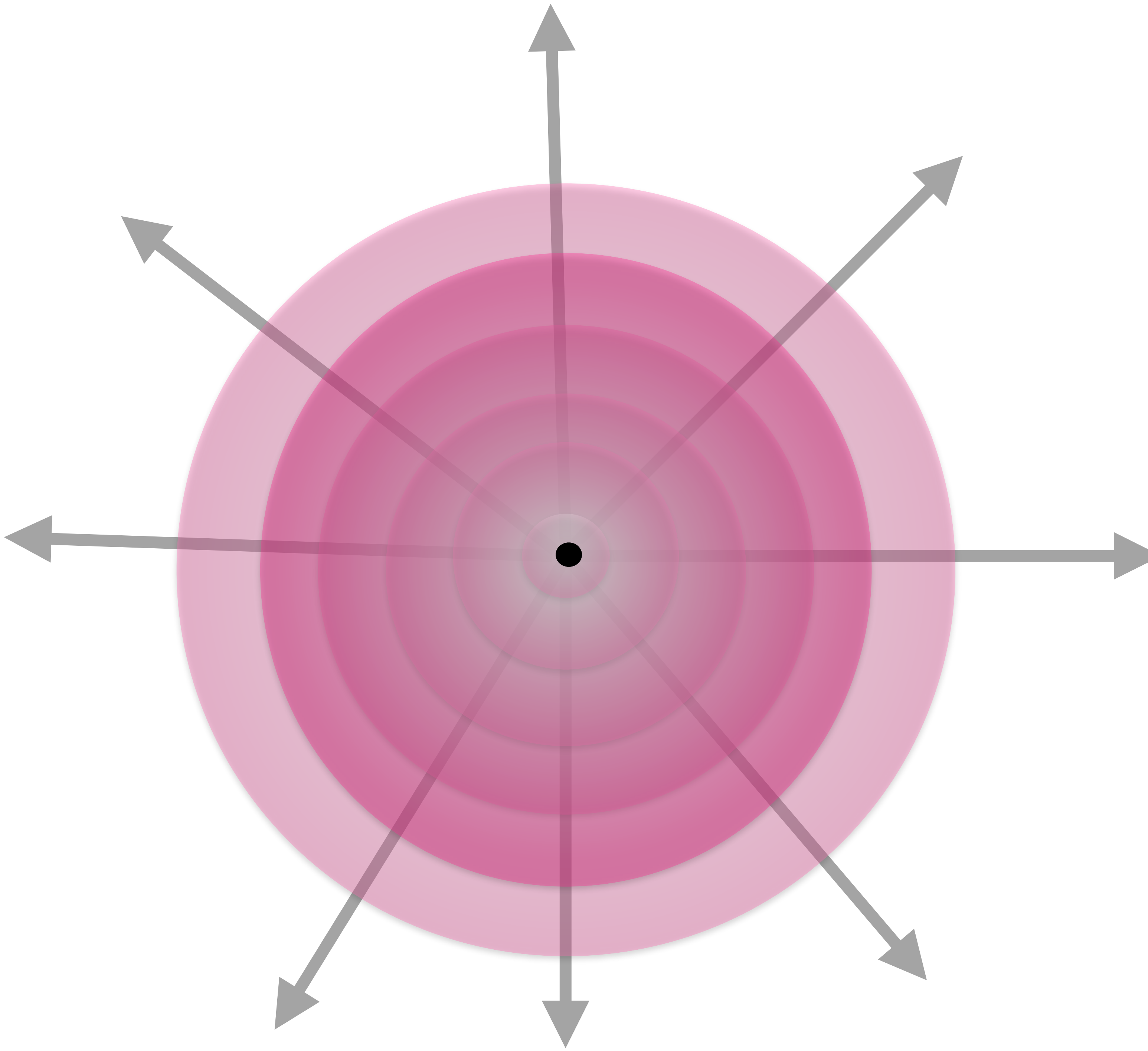
The general solution of this equation can be written as

$$\boxed{f(\alpha x + \beta y + \gamma z - vt)}$$

$$\alpha^2 + \beta^2 + \gamma^2 = 1$$

Spherical waves

- Waves emitting from a point source.
- Wave fronts are concentric spheres.
- It is called an isotropic source.



Spherical waves

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

- Spherical coordinates: r, θ, ϕ
- Laplacian in r, θ, ϕ

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial^2}{\partial\phi^2}\right)$$

- Spherical waves are radially symmetric

$$f(r, \theta, \phi) = f(r)$$ independent of angular coordinates

$$\boxed{\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)}$$

Spherical waves

$$\nabla^2 f(r) = \frac{1}{r}\frac{\partial^2}{\partial r^2}[rf(r)]$$

- Finally, we can write the wave equation as:

$$\frac{\partial^2}{\partial r^2}[rf(r)] = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}[rf(r)]$$

$$\boxed{rf(r,t) = g(r - vt)}$$

We have now reduced to a one dimensional wave equation.

$$f(r,t) = \frac{g(r - vt)}{r}$$

Represents the spherical waves progressing radially outward from the source.

Spherical waves

- Harmonic spherical waves

$$f(r,t) = \frac{A}{r} \times \cos(kr \pm \omega t)$$

OR

$$f(r,t) = \frac{A}{r} \times e^{i(kr \pm \omega t)}$$

- The key difference from the normal plane wave is that the amplitude is position dependent for a spherical wave: *A/r*.

Plane waves

$\longrightarrow r$