Lecture 3.

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Contents

Literature:

1. Optics, (Eugene Hecht and A. R. Ganesan)
2. Optical Physics, (A. Lipson, S. G. Lipson and H. Lipson)
3. The Optics of Life, (Sönke Johnsen)
4. Modern Optics, (Grant R. Fowles)

Course Contents

1. **Nature of light (waves and particles)**
2. **Maxwells equations and wave equation**
3. **Poynting vector**
4. **Polarization of light**
5. **Law of reflection and snell's law**
6. **Total Internal Reflection and Evanescent waves**
7. **Concept of coherence and interference**
8. **Young's double slit experiment**
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Coherence and interference

Optical interference is based on the superposition principle.
(This is because the Maxwell's equations are linear differential equations.)

The electric field at a point in vacuum:

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \mathbf{E}_4 + ...$$

is the vector sum of that from the different sources.

The same is true for magnetic fields.

Remark: This may not be generally true, deviations from linear superposition is the study of ***nonlinear optical phenomena or simply non-linear optics.***

Coherence and interference

Consider two harmonic, monochromatic, linearly polarised waves

$$\mathcal{E}_1 = \mathbf{E}_1 \exp\left[i(\mathbf{k}_1 \cdot \mathbf{r} - \omega t + \phi_1)\right]$$

$$\mathcal{E}_2 = \mathbf{E}_2 \exp\left[i(\mathbf{k}_2 \cdot \mathbf{r} - \omega t + \phi_2)\right]$$

If the phase difference $\phi_1 - \phi_2$ is constant, the two sources are said to be ***mutually coherent.***

The irradiance or the intensity at a point is provided by

$$I = |\mathcal{E}_1 + \mathcal{E}_2|^2$$

$$= |\mathbf{E}_1|^2 + |\mathbf{E}_2|^2 + 2\mathbf{E}_1 \cdot \mathbf{E}_2 \cos\theta$$

$$\theta = \mathbf{k}_1 \cdot \mathbf{r} - \mathbf{k}_2 \cdot \mathbf{r} + \phi_1 - \phi_2$$

interference term

(all the fun is due to this!)

Coherence and interference

$$I = |\mathbf{E}_1|^2 + |\mathbf{E}_2|^2 + 2\mathbf{E}_1 \cdot \mathbf{E}_2 \cos\theta$$

interference term
(all the fun is due to this!)

$$\theta = \mathbf{k}_1 \cdot \mathbf{r} - \mathbf{k}_2 \cdot \mathbf{r} + \phi_1 - \phi_2$$

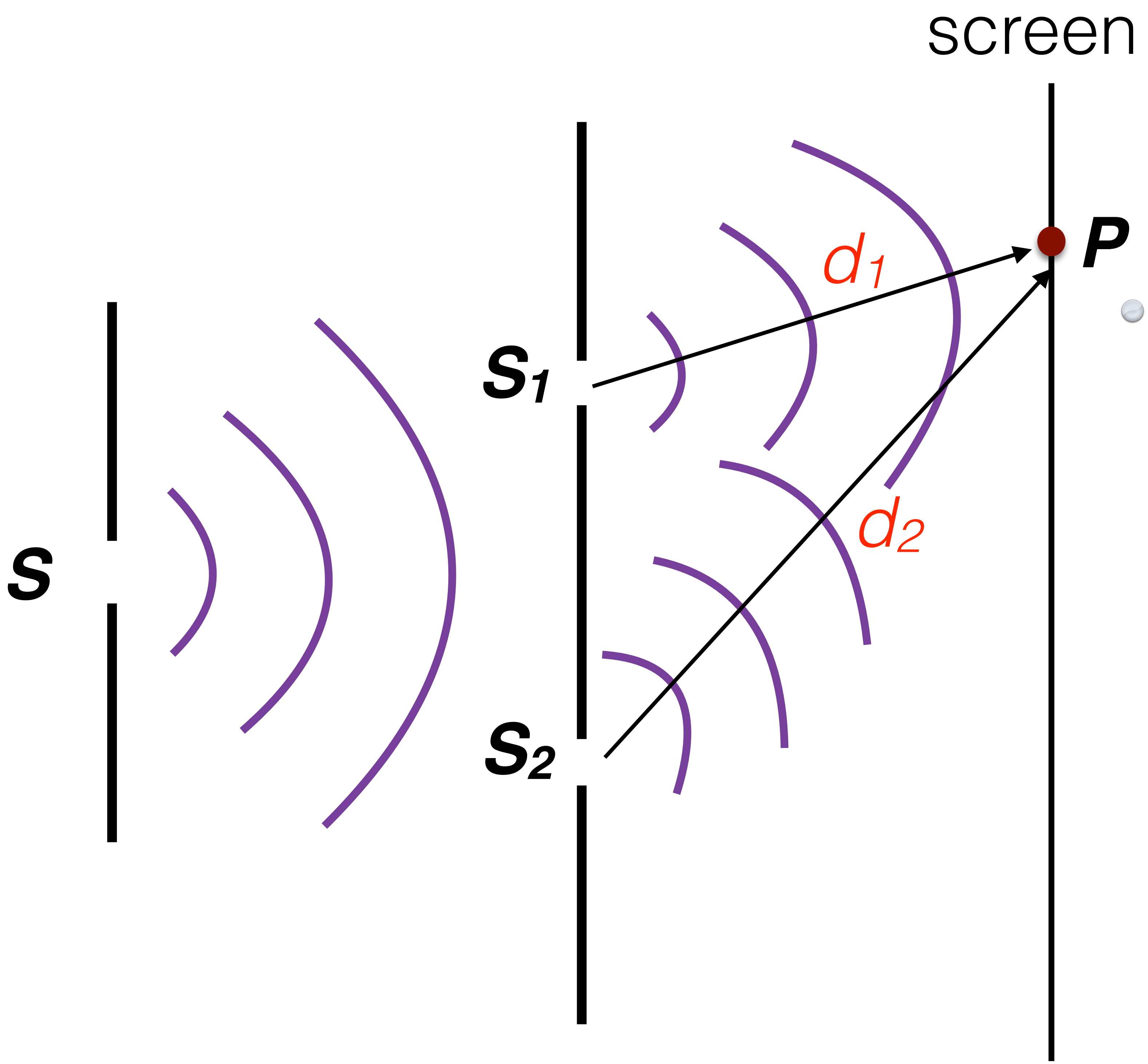
This indicates that I can greater than or less than I_1+I_2.

Since θ depends on position, periodic spatial variations occur (fringes).

- What happens if two waves are mutually incoherent?
- What happens if the polarisations are mutually orthogonal?

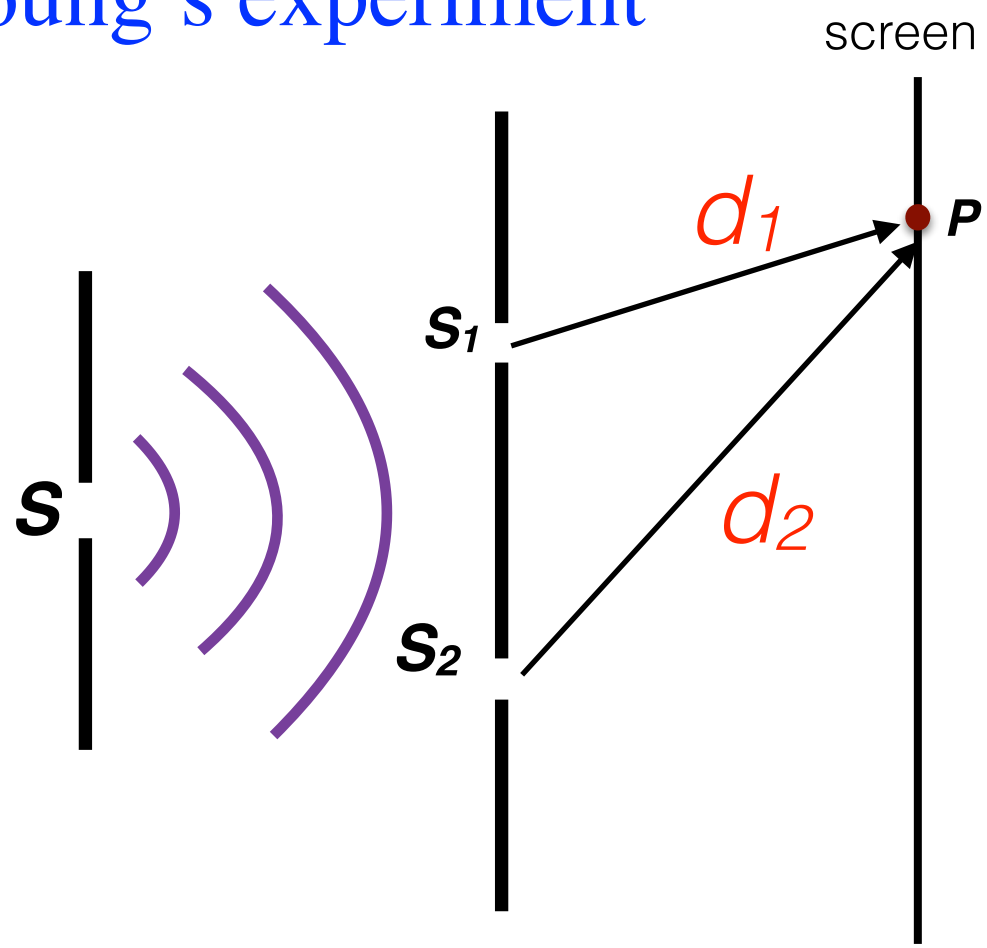
Young's experiment

performed by Thomas young in 1802.



- In this setup, what is so crucial to have the interference pattern?

Young's experiment



All you have to know is the phase difference of waves arriving at P from the two point sources (spherical waves).

$$\Delta\phi = k(d_2 - d_1)$$

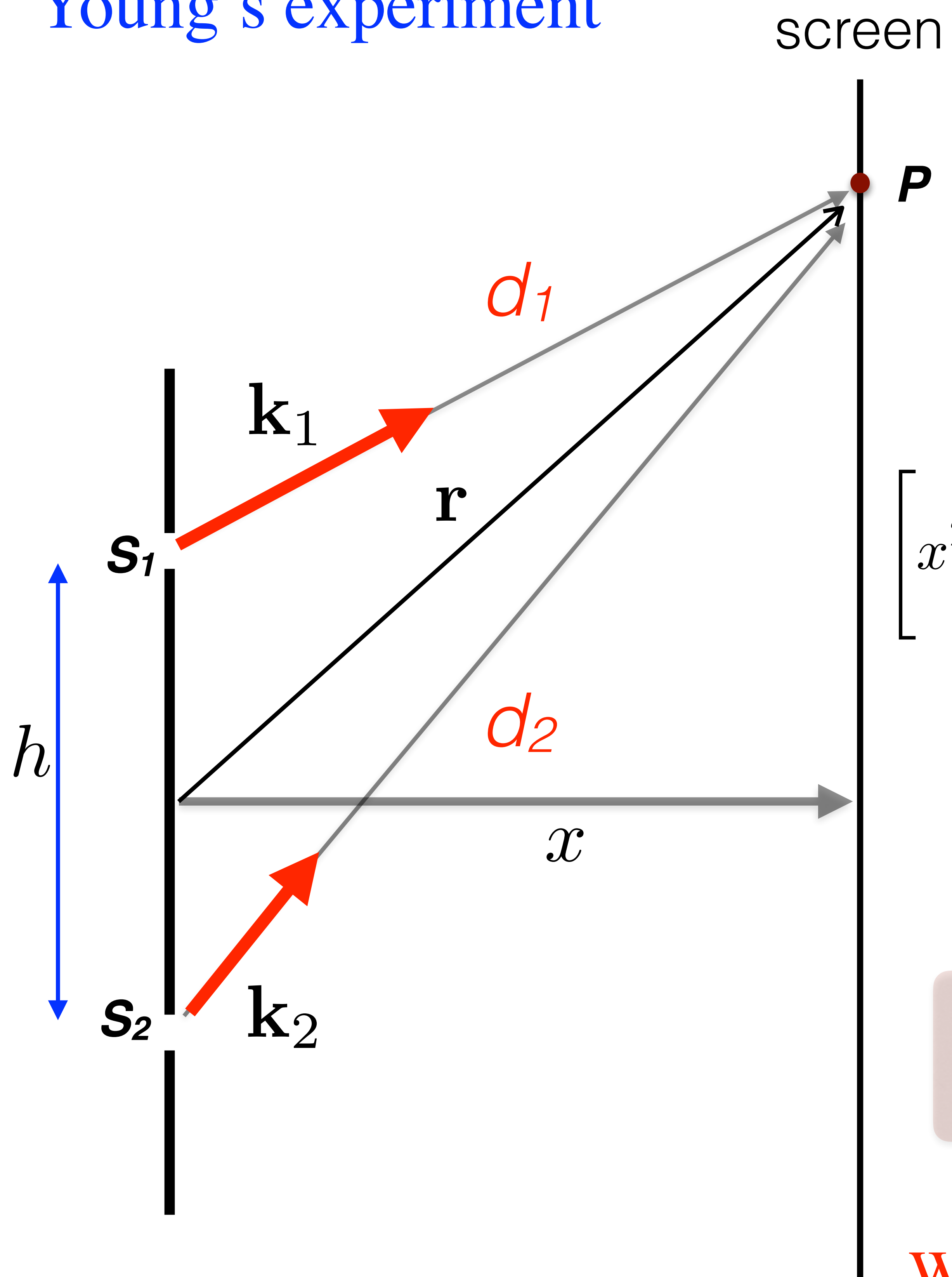
Constructive interference

$$k(d_2 - d_1) = \pm 2n\pi$$

$$\boxed{|d_2 - d_1| = n\lambda}$$

path difference is equal to the integer number of wave lengths.

Young's experiment



$$\boxed{|d_2 - d_1| = n\lambda}$$

$$\left[x^2 + \left(y + \frac{h}{2}\right)^2\right]^{1/2} - \left[x^2 + \left(y - \frac{h}{2}\right)^2\right]^{1/2} = n\lambda$$

$$x \gg y, h$$

$$\boxed{\frac{yh}{x} = n\lambda}$$

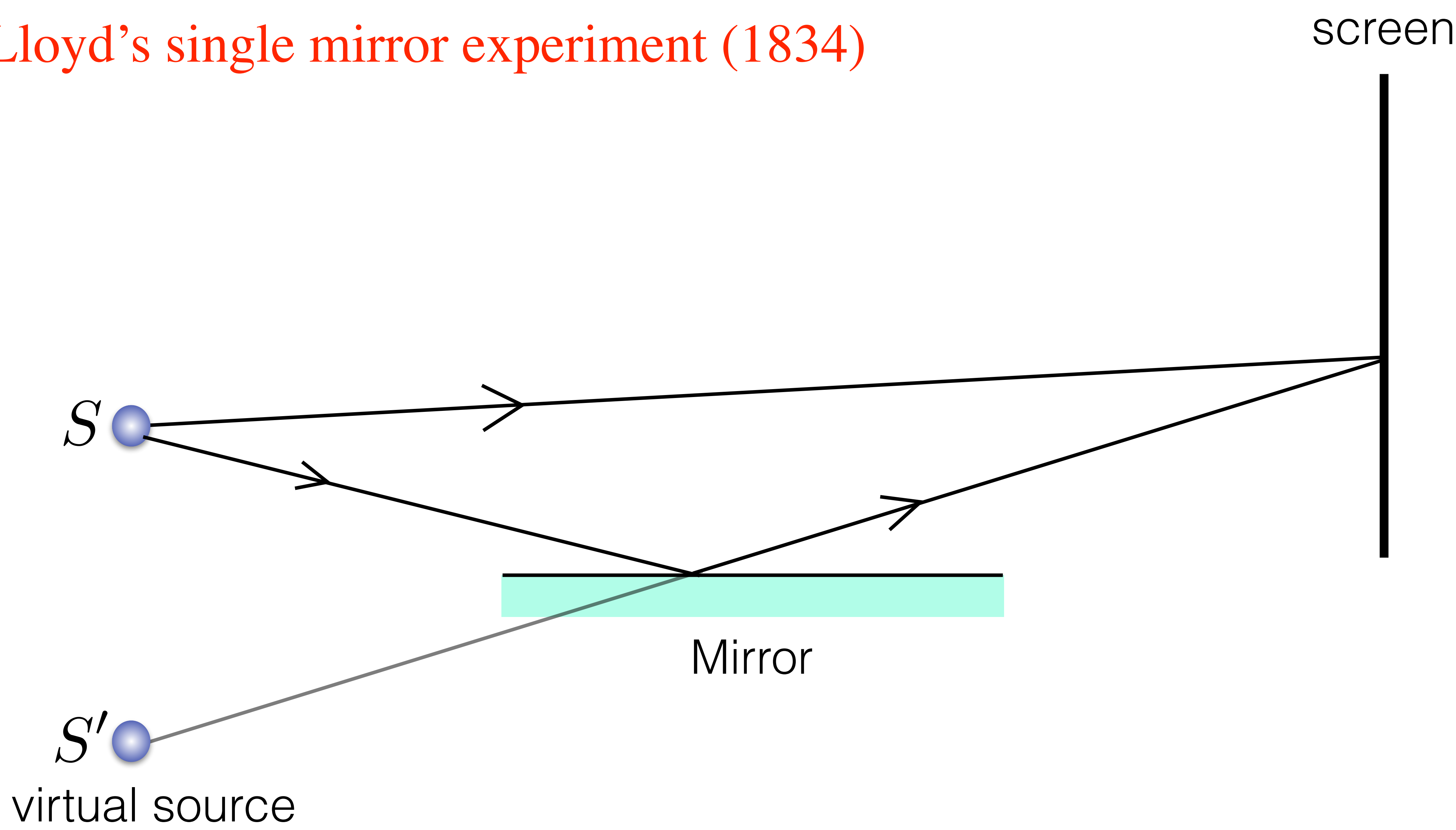
This tells you where the bright fringes are on the screen.

What is the central fringe, bright or dark?

Can you displace the fringes on the screen?

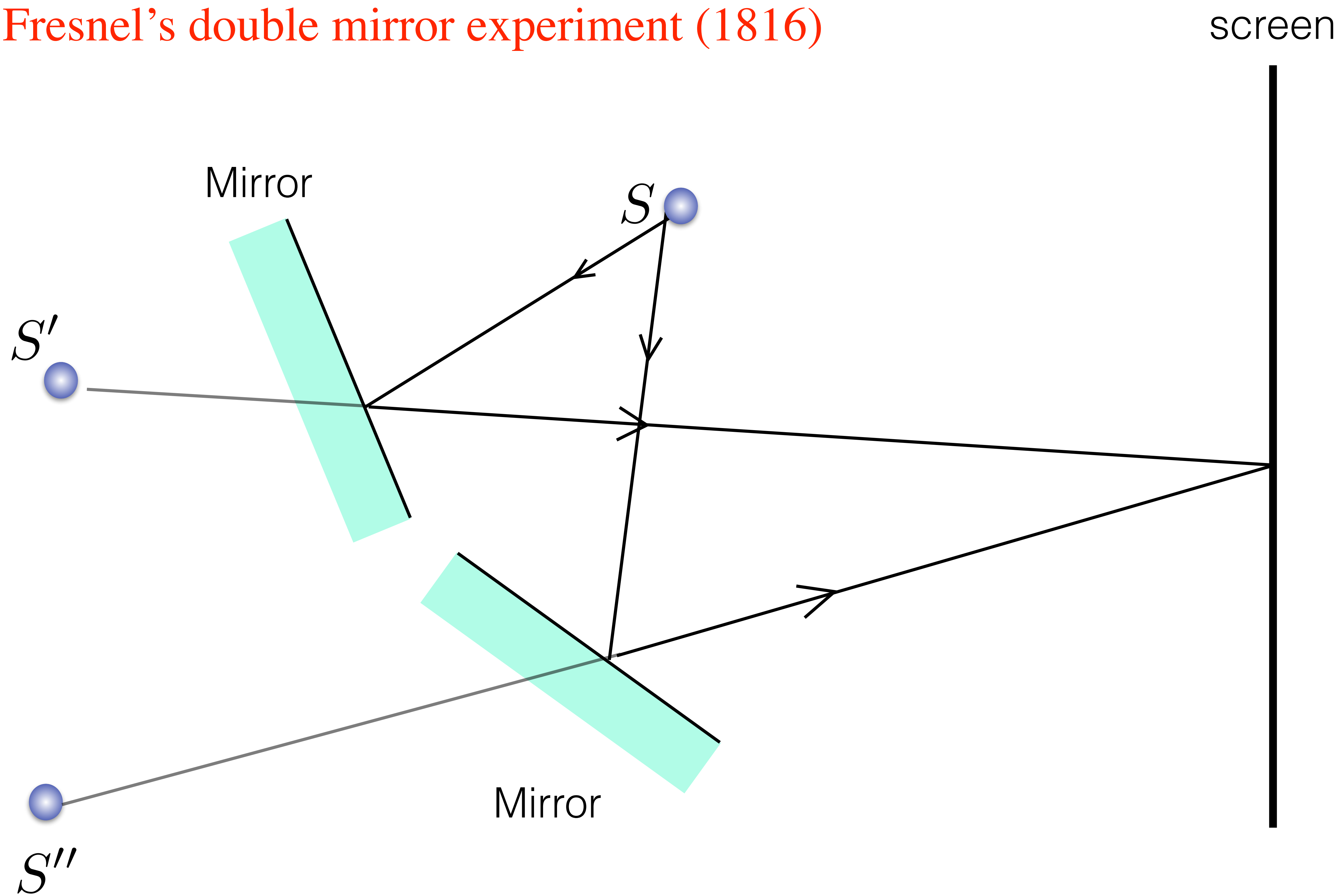
Alternative methods

Lloyd's single mirror experiment (1834)



Alternative methods

Fresnel's double mirror experiment (1816)



Home work: Fresnel's bi-prism experiment.

Basic theory of coherence

The irradiance or the intensity at a point is provided by

$$I = |\mathcal{E}_1 + \mathcal{E}_2|^2$$

$$= |\mathbf{E}_1|^2 + |\mathbf{E}_2|^2 + 2\mathbf{E}_1 \cdot \mathbf{E}_2 \cos\theta$$

This equation is based on the following assumptions about the two fields:

1. ***completely*** coherent
2. monochromatic
3. constant in amplitude

In reality the phase and the amplitude vary with time in a random manner.

Basic theory of coherence

$$I = \langle \mathbf{E} \cdot \mathbf{E}^* \rangle$$

$$= \langle |\mathbf{E}_1|^2 + |\mathbf{E}_2|^2 + 2\mathrm{Re}(\mathbf{E}_1 \cdot \mathbf{E}_2^*) \rangle$$

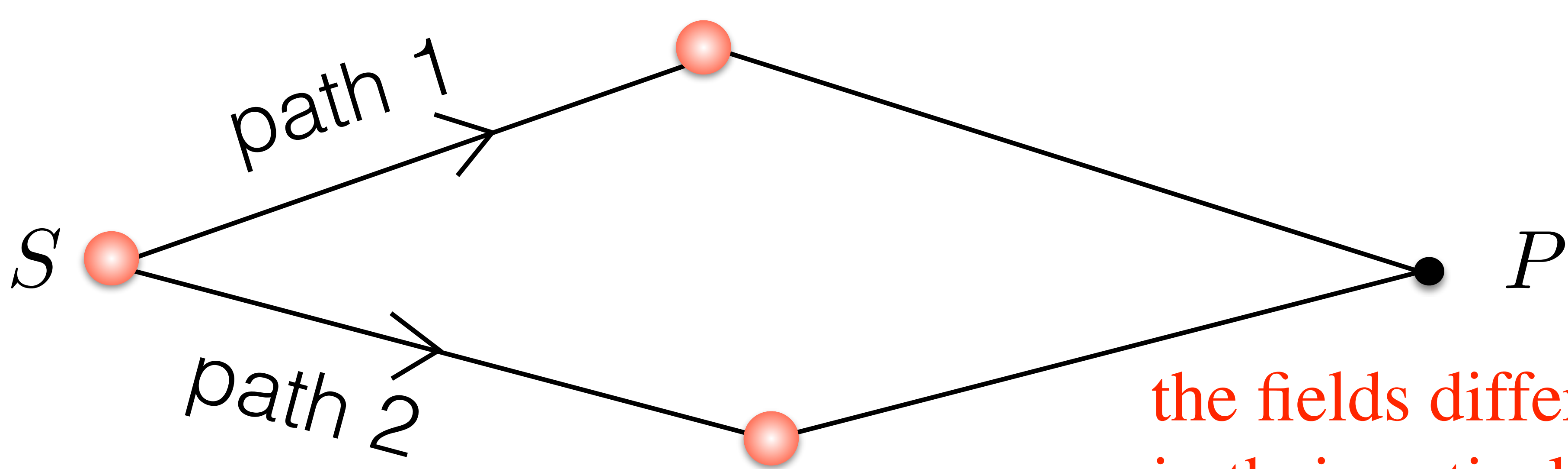
In reality the phase and the amplitude vary with time in a random manner.

Assuming the average quantities are stationary and for the same polarization for two fields:

Time average

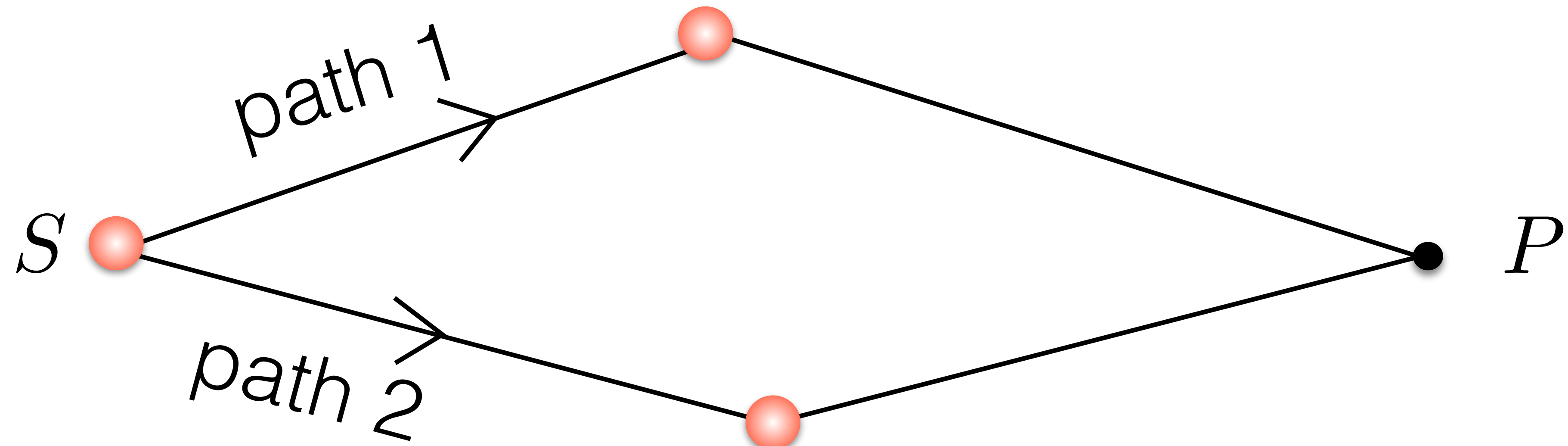
$$\langle f \rangle = \lim_{T\to\infty} \frac{1}{T} \int_0^T f(t)dt$$

$$I = I_1 + I_2 + 2\mathrm{Re}\langle E_1 E_2^* \rangle$$



the fields differ due to difference in their optical paths.

Basic theory of coherence



The time difference between two paths: τ

The interference term can be written as

$$2\mathrm{Re}\langle E_1(t)E_2^*(t+\tau)\rangle = 2\mathrm{Re}[\Gamma_{12}(\tau)]$$

Mutual coherence function

$$\Gamma_{11}(\tau) = ??? \qquad \Gamma_{11}(0) = ??? \qquad \Gamma_{22}(0) = ???$$

Auto-correlation or self coherence functions

Basic theory of coherence

We define the normalised coherence function:

$$\gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{\sqrt{\Gamma_{11}(0)\Gamma_{22}(0)}} = \frac{\Gamma_{12}(\tau)}{\sqrt{I_1 I_2}}$$

in general a complex periodic function.

complex degree of partial coherence

The intensity at a particular point in space now reads as

$$\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2} Re[\gamma_{12}(\tau)]}$$

$$0 \leq |\gamma_{12}(\tau)| \leq 1$$

$|\gamma_{12}(\tau)| = 0$ (completely incoherent)

$0 < |\gamma_{12}(\tau)| < 1$ (partially coherent)

$|\gamma_{12}(\tau)| = 1$ (completely coherent)

Basic theory of coherence

For an interference pattern (fix the source characterised by γ_{12}), the intensity varies between I_{max} and I_{min}.

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}|\gamma_{12}|$$

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}|\gamma_{12}|$$

We define the fringe visibility or contrast:

$$\mathcal{V} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{I_1 I_2}|\gamma_{12}|}{I_1 + I_2}$$

If $I_1 = I_2$,

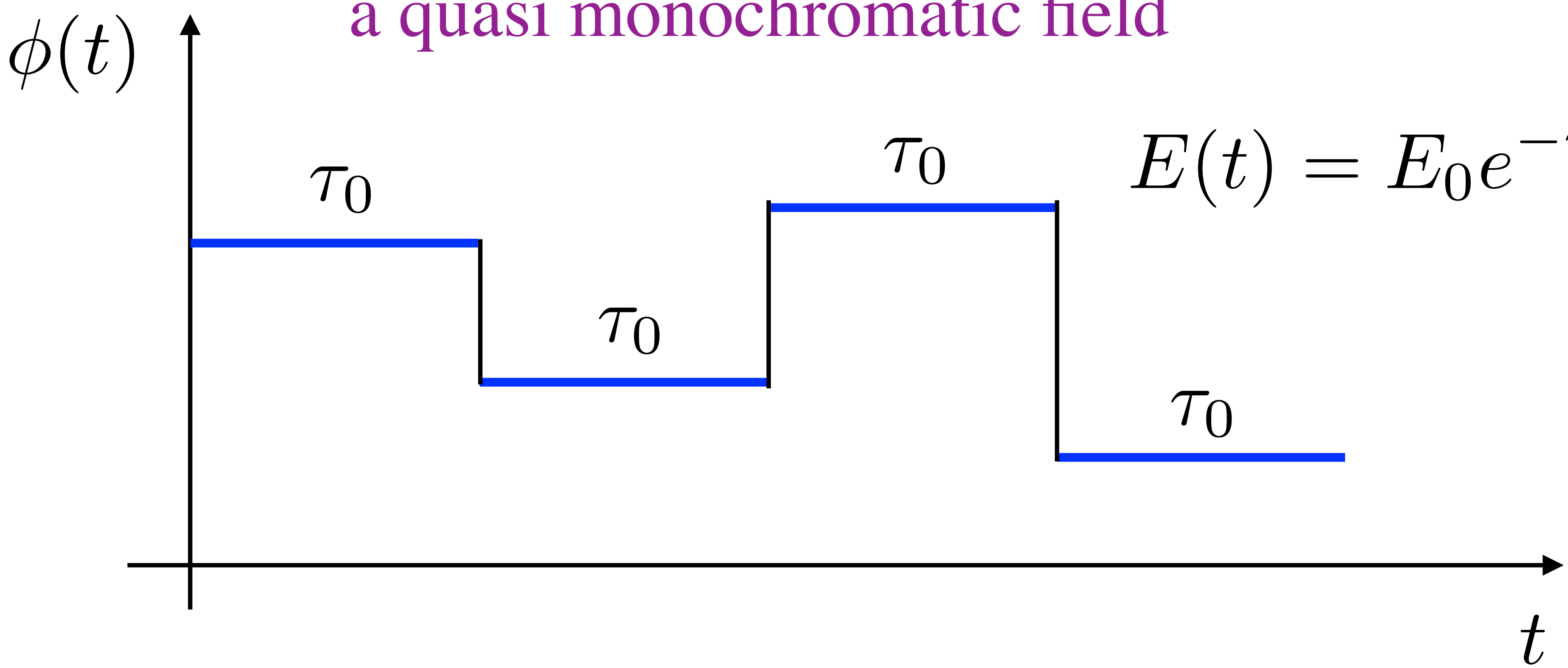
$$\mathcal{V} = |\gamma_{12}|$$

is zero if the fields are incoherent, and maximum if they are coherent.

Coherence time and coherence length

How the degree of partial coherence is related to the source characteristics?

a quasi monochromatic field



$$E(t) = E_0 e^{-i\omega t + i\phi(t)}$$

The phase of the field is making some random jumps.

Suppose I divide the field into two equal fields and now the interference pattern will be governed by

$$\gamma(\tau) = \frac{\langle E(t) E^*(t+\tau) \rangle}{\langle |E|^2 \rangle}$$ self coherence function

Coherence time and coherence length

$$\gamma(\tau) = \frac{\langle E(t)E^*(t+\tau)\rangle}{\langle |E|^2\rangle}$$

self coherence function

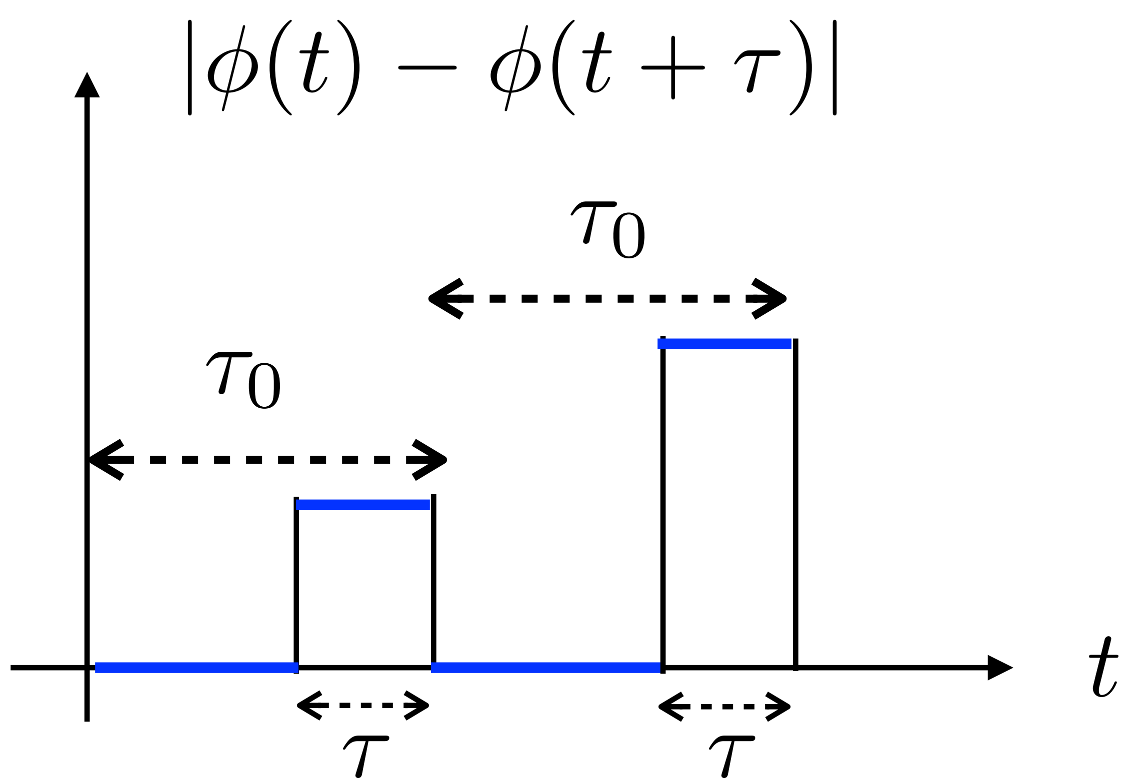
$$E(t) = E_0 e^{-i\omega t + i\phi(t)}$$

$$\gamma(\tau) = \left\langle e^{i\omega\tau} e^{i[\phi(t)-\phi(t+\tau)]} \right\rangle$$

$$= e^{i\omega\tau} \lim_{T\to\infty} \frac{1}{T}\int_0^T e^{i[\phi(t)-\phi(t+\tau)]} dt$$

Take $\tau < \tau_0$

$$\frac{1}{\tau_0}\int_0^{\tau_0} e^{i[\phi(t)-\phi(t+\tau)]} dt = ???$$



Coherence time and coherence length

$$\frac{1}{\tau_0}\int_0^{\tau_0} e^{i[\phi(t)-\phi(t+\tau)]}dt = \frac{\tau_0-\tau}{\tau_0} + \frac{\tau}{\tau_0}e^{i\Delta}$$

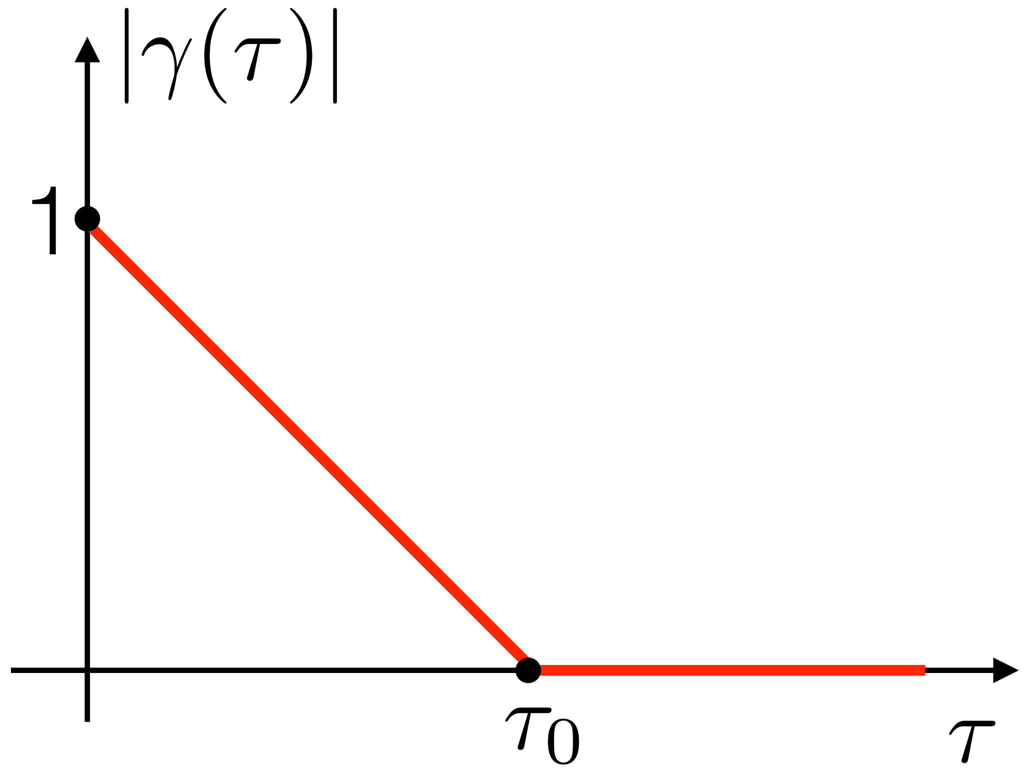
- First term is same for all intervals.
- The second term averages to zero over long time.

$$\gamma(\tau) = \left(1-\frac{\tau}{\tau_0}\right)e^{i\omega\tau} \qquad (\tau < \tau_0)$$

$$= 0 \qquad (\tau \geq \tau_0)$$

Plot $|\gamma(\tau)|$ vs τ.

Coherence time and coherence length



τ_0 is the **coherence time**.

- provides the fringe visibility
  (or the coherence of a quasi-monochromatic source)

- the path difference between two beams must not exceed

$$l_c = c\tau_0$$

to observe the interference fringes. This length is called the **coherence length** or **the length of an uninterrupted wave.**

Coherence time and coherence length

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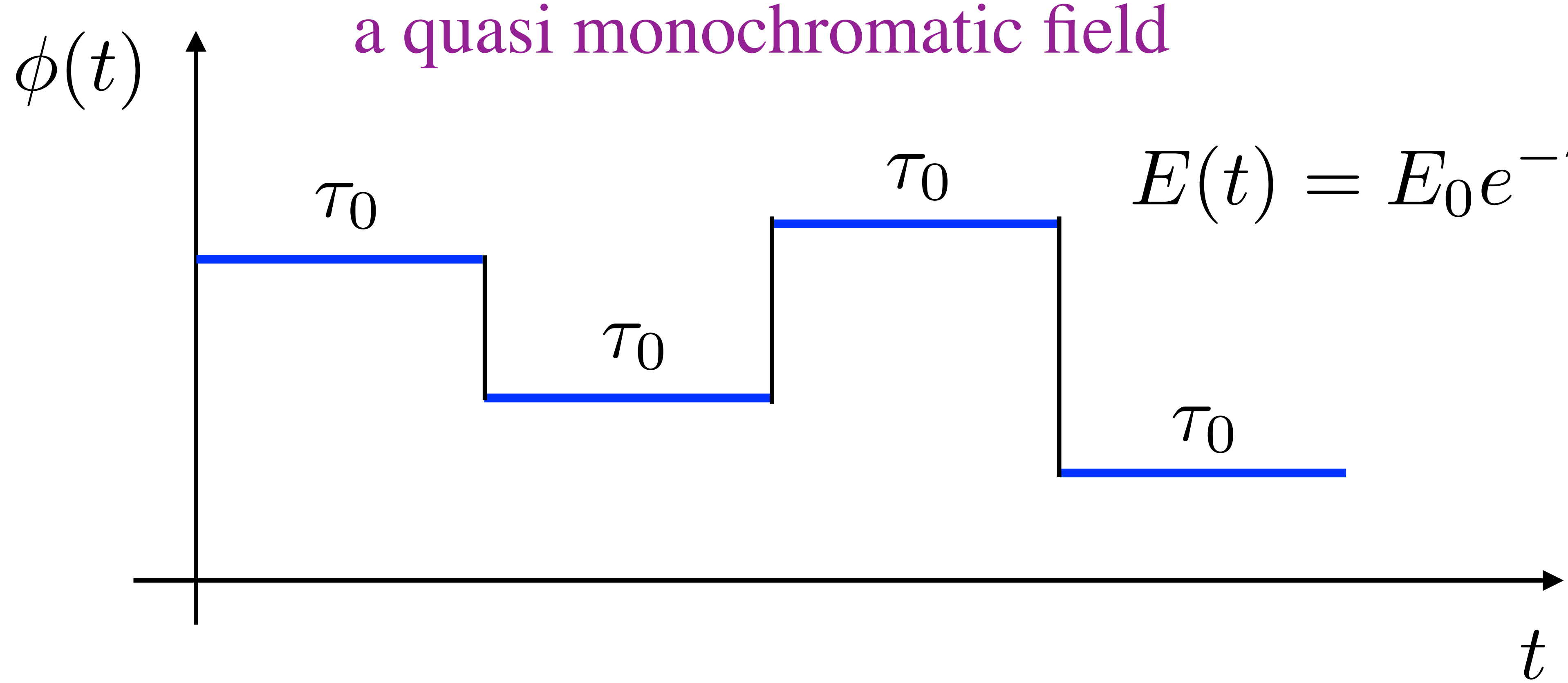
a quasi monochromatic field



$$E(t) = E_0 e^{-i\omega t + i\phi(t)}$$

In reality there is a statistical distribution for coherence lengths (or time).

A finite wave train

$$f(t) = e^{-i\omega_0 t} \qquad \text{for} \qquad -\frac{\tau_0}{2} < t < \frac{\tau_0}{2}$$

$$= 0 \qquad \text{otherwise}$$

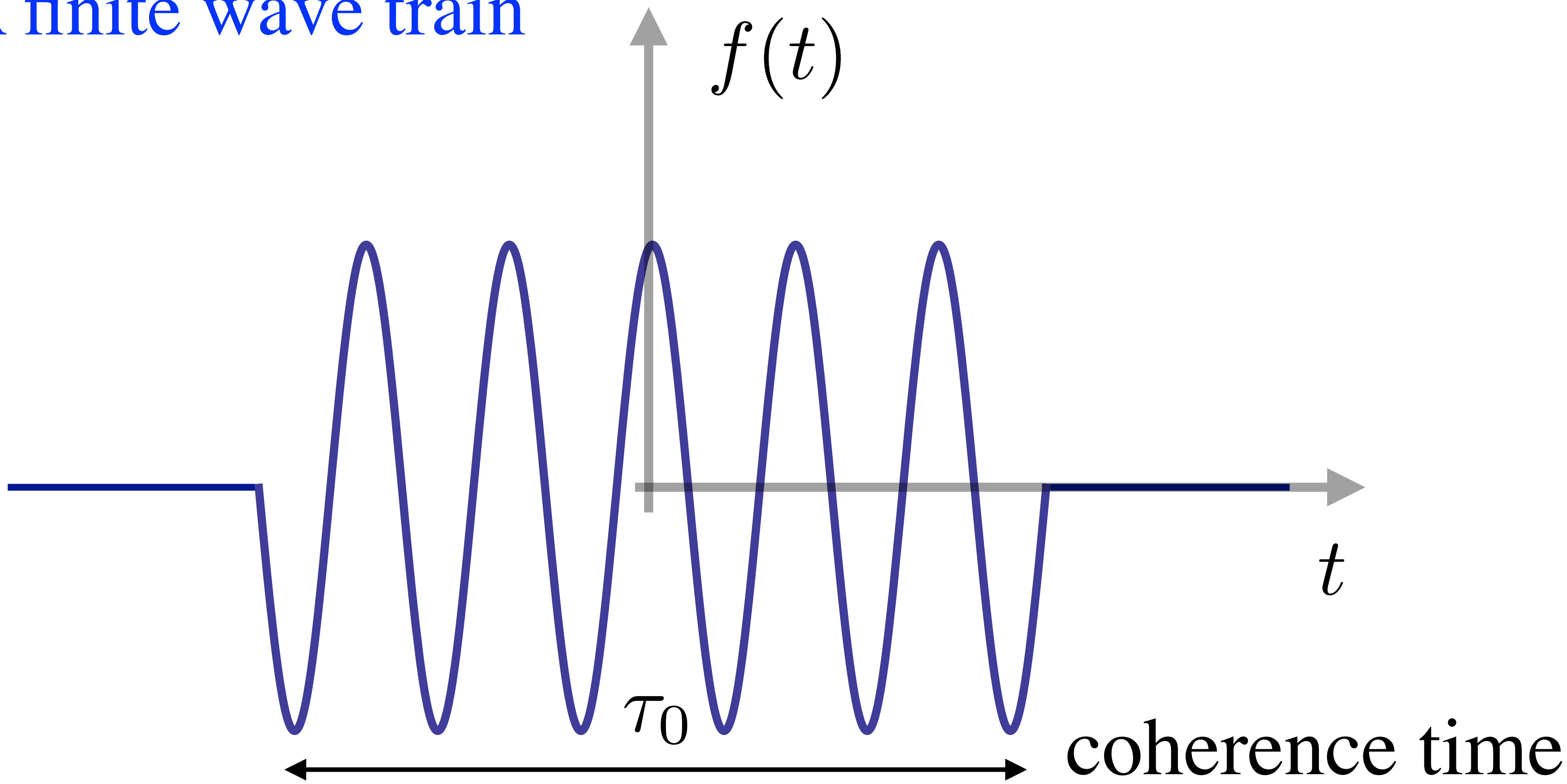
Obtain its Fourier transform:

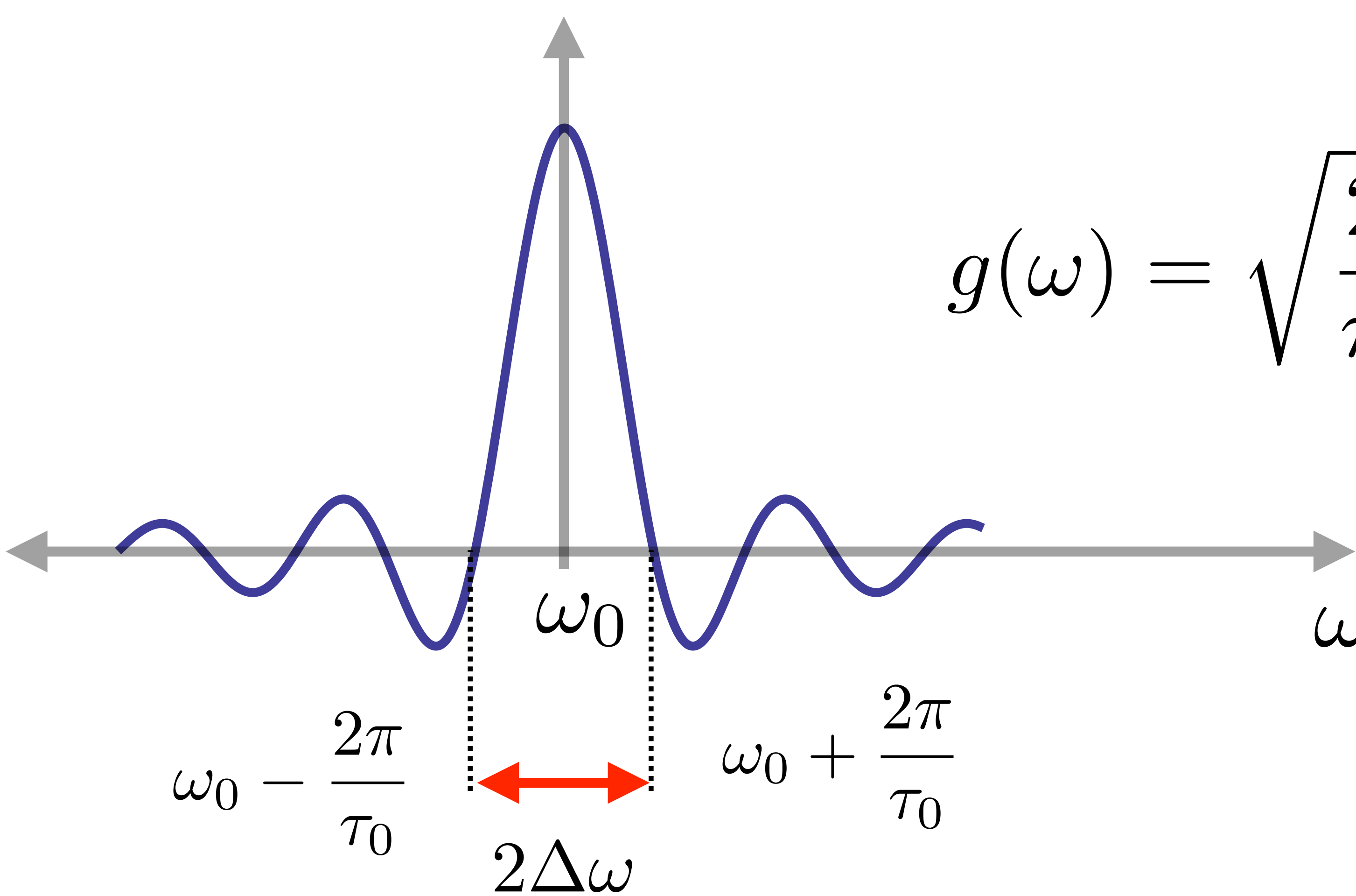
$$\boxed{\begin{aligned} f(t) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(\omega) e^{-i\omega t} d\omega \\ g(\omega) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt \end{aligned}}$$

$$g(\omega) = \sqrt{\frac{2}{\pi}} \frac{\sin[(\omega - \omega_0)\tau_0/2]}{\omega - \omega_0}$$

$$\boxed{\begin{gathered} G(\omega) = |g(\omega)|^2 \\ \textit{Power spectrum} \end{gathered}}$$

A finite wave train





$$g(\omega) = \sqrt{\frac{2}{\pi}} \frac{\sin[(\omega - \omega_0)\tau_0/2]}{\omega - \omega_0}$$

$$\Delta\omega = \frac{2\pi}{\tau_0}$$

A finite wave train

$$\Delta\omega = \frac{2\pi}{\tau_0} \qquad\qquad \Delta\nu = \frac{1}{\tau_0}$$

In other words *"the frequency width or line width"* gives you the corresponding *coherence time* and also the *coherence length*.

Exercise: Show that the coherence length can be expressed as,

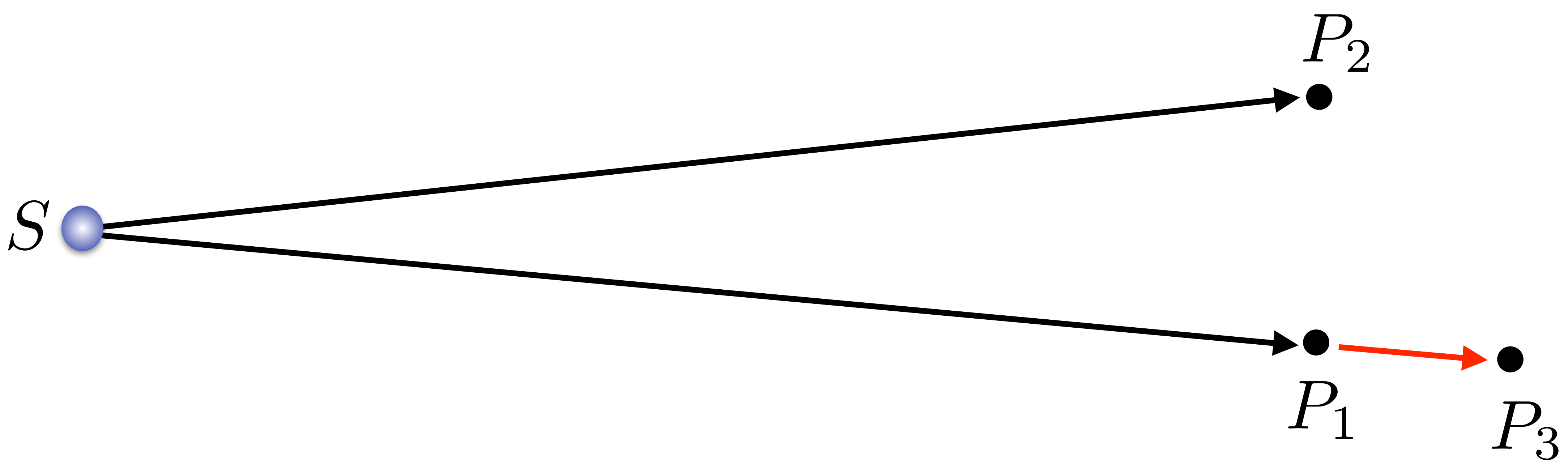
$$l_c = \frac{\lambda^2}{\Delta\lambda}$$

Ordinary light sources $\quad \Delta\lambda \sim 5000\ \text{Å}$

$$l_c \sim 2mm$$

What if we take a LASER fields?

Spatial coherence



The coherence between the fields at P_1 and P_3 measures the *longitudinal spatial coherence* of the field.

The coherence between the fields at P_1 and P_2 measures the *transversal spatial coherence* of the field.

Interference Methods: Classifications

Interference by

1. division of wave front.

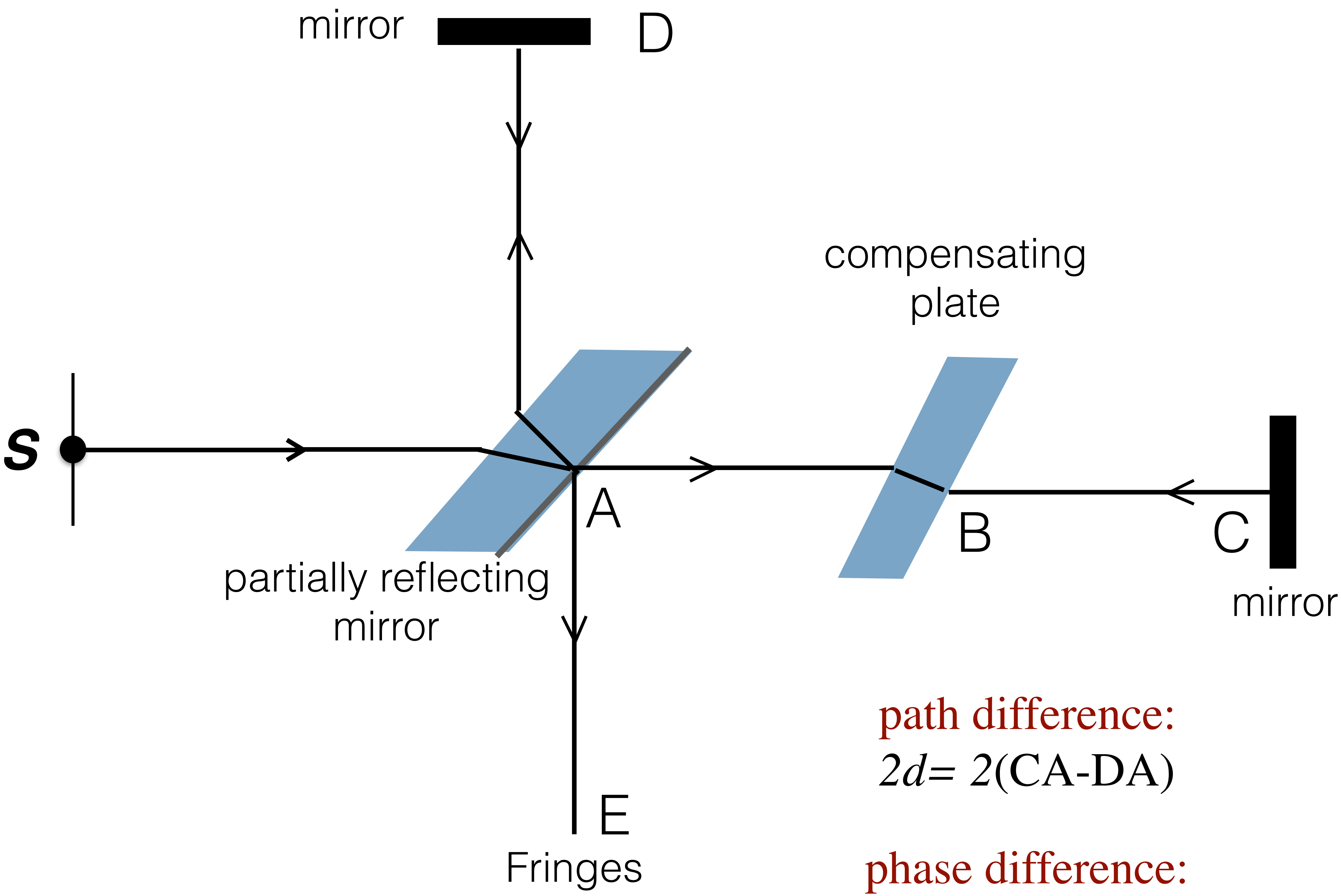
   A single point like source emitting waves in different directions. Theses waves are then brought together by means of mirrors, prisms and lenses.

2. division of amplitude.

   A single beam (or a wave) of light is divided into two or more beams by partial reflection.

   e.g. Michelson interferometer

Michelson Interferometer (1880)



path difference:

$2d = 2(\text{CA-DA})$

phase difference:

$$\Delta\phi = k \times 2d = \frac{2\pi}{\lambda} \times 2d$$