Lecture 4.

Rejish Nath

Contents

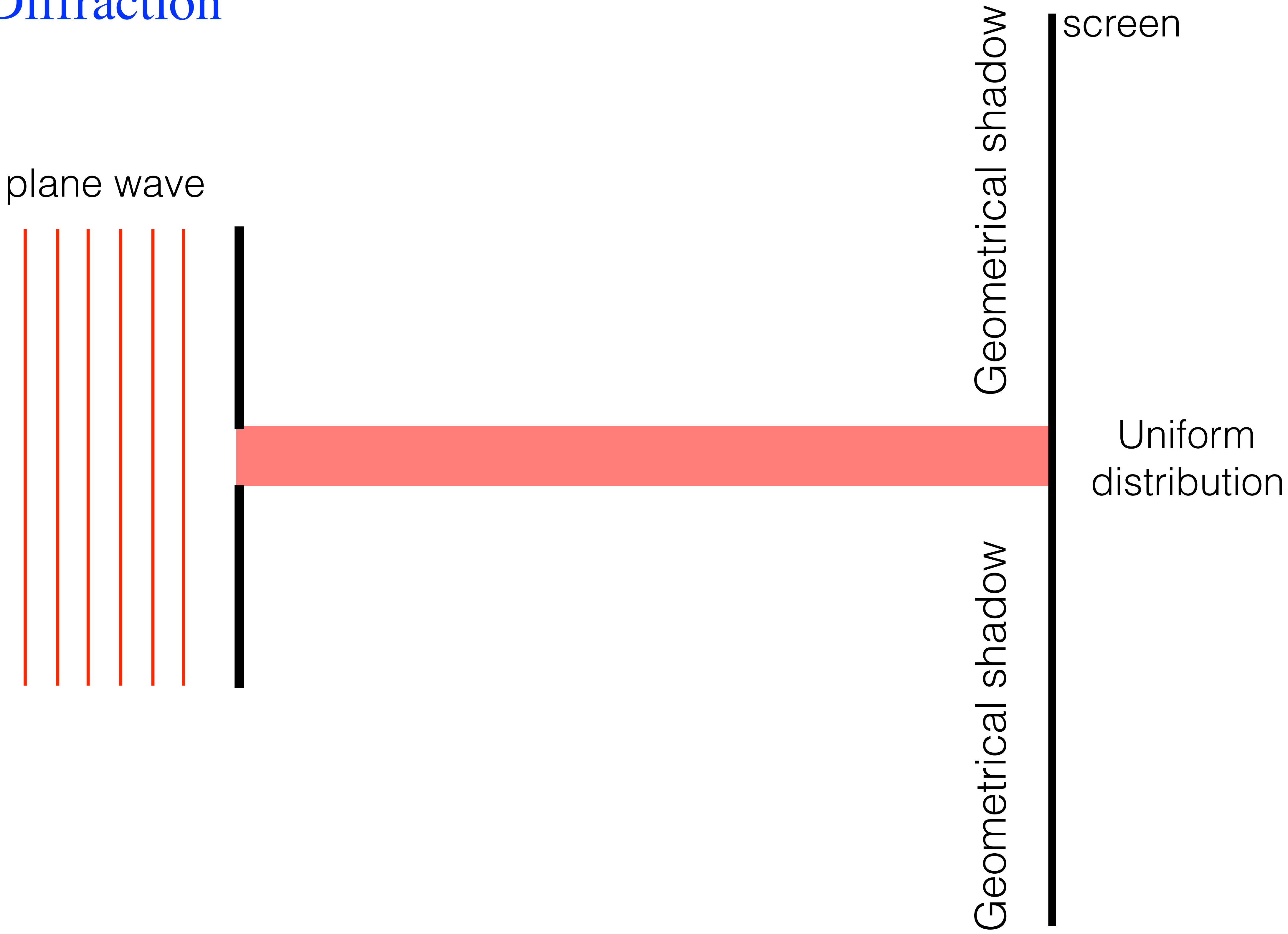
Literature:

1. Optics, (Eugene Hecht and A. R. Ganesan)
2. Optical Physics, (A. Lipson, S. G. Lipson and H. Lipson)
3. Optics, (A. Ghatak)

Course Contents

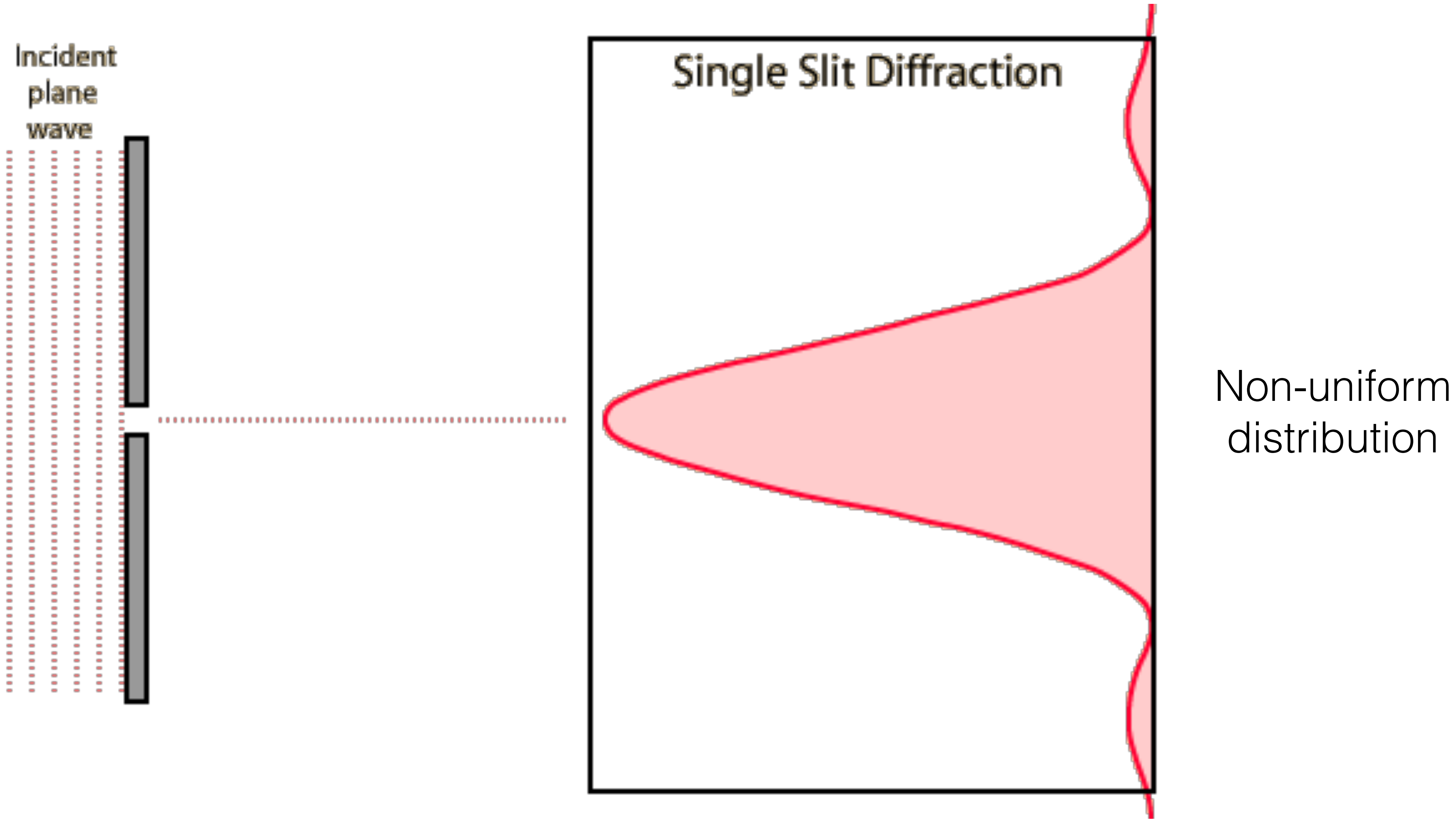
1. **Nature of light (waves and particles)**
2. **Maxwells equations and wave equation**
3. **Poynting vector**
4. **Polarization of light**
5. **Law of reflection and snell's law**
6. **Total Internal Reflection and Evanescent waves**
7. **Concept of coherence and interference**
8. **Young's double slit experiment**
9. **Single slit, N-slit Diffraction**
10. Grating, Birefringence, Retardation plates
11. **Fermat's Principle**
12. Optical instruments
13. Human Eye
14. Spontaneous and stimulated emission
15. Concept of Laser

Diffraction



This is what you expect according to geometrical optics (light travels in straight lines)

Diffraction



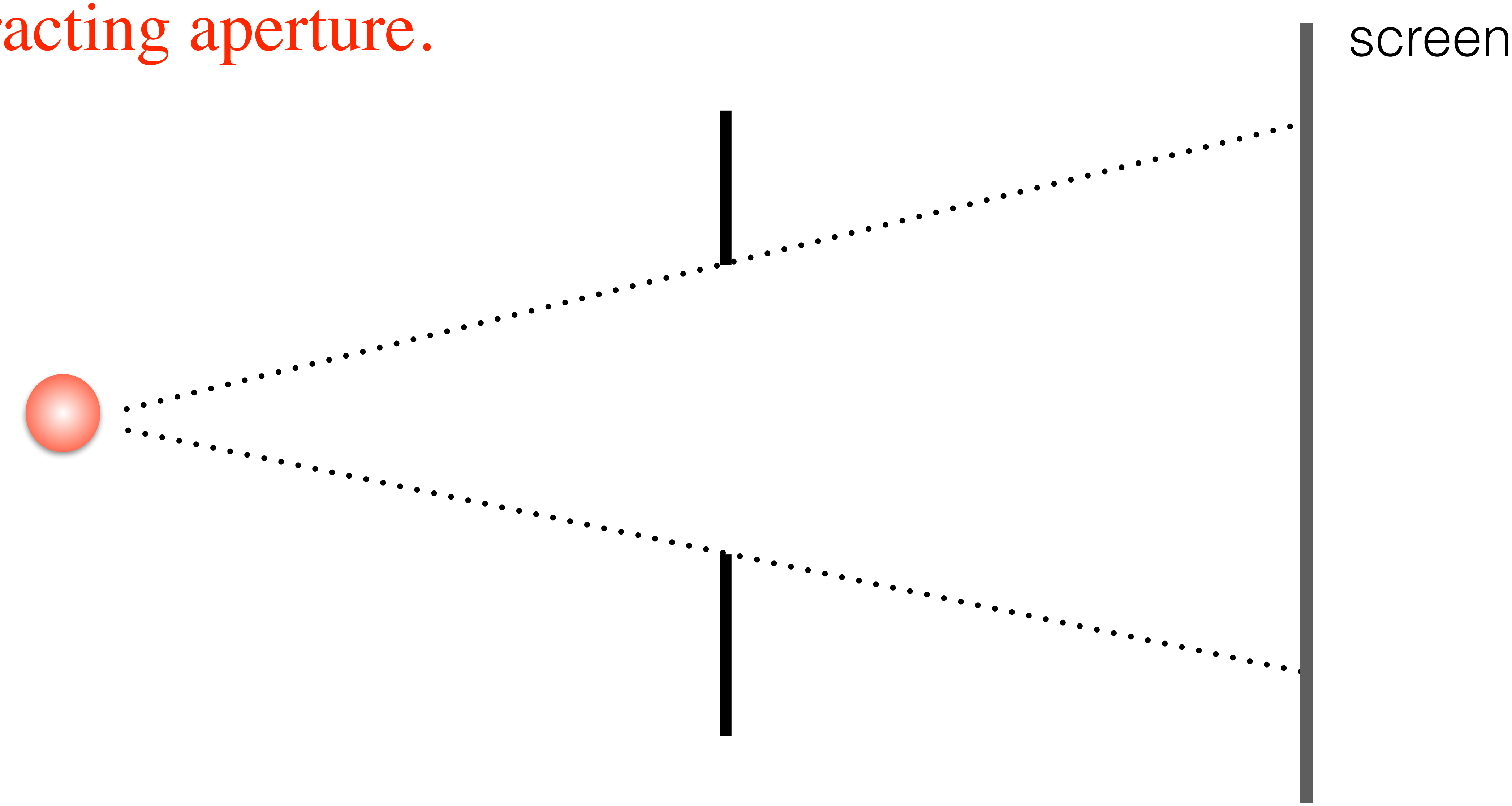
If the slit width is comparable to the wavelength of the light used.

Diffraction

Divided into two categories

Fresnel and Franhofer diffractions

- Fresnel: Both source and screen are at finite distance from the diffracting aperture.

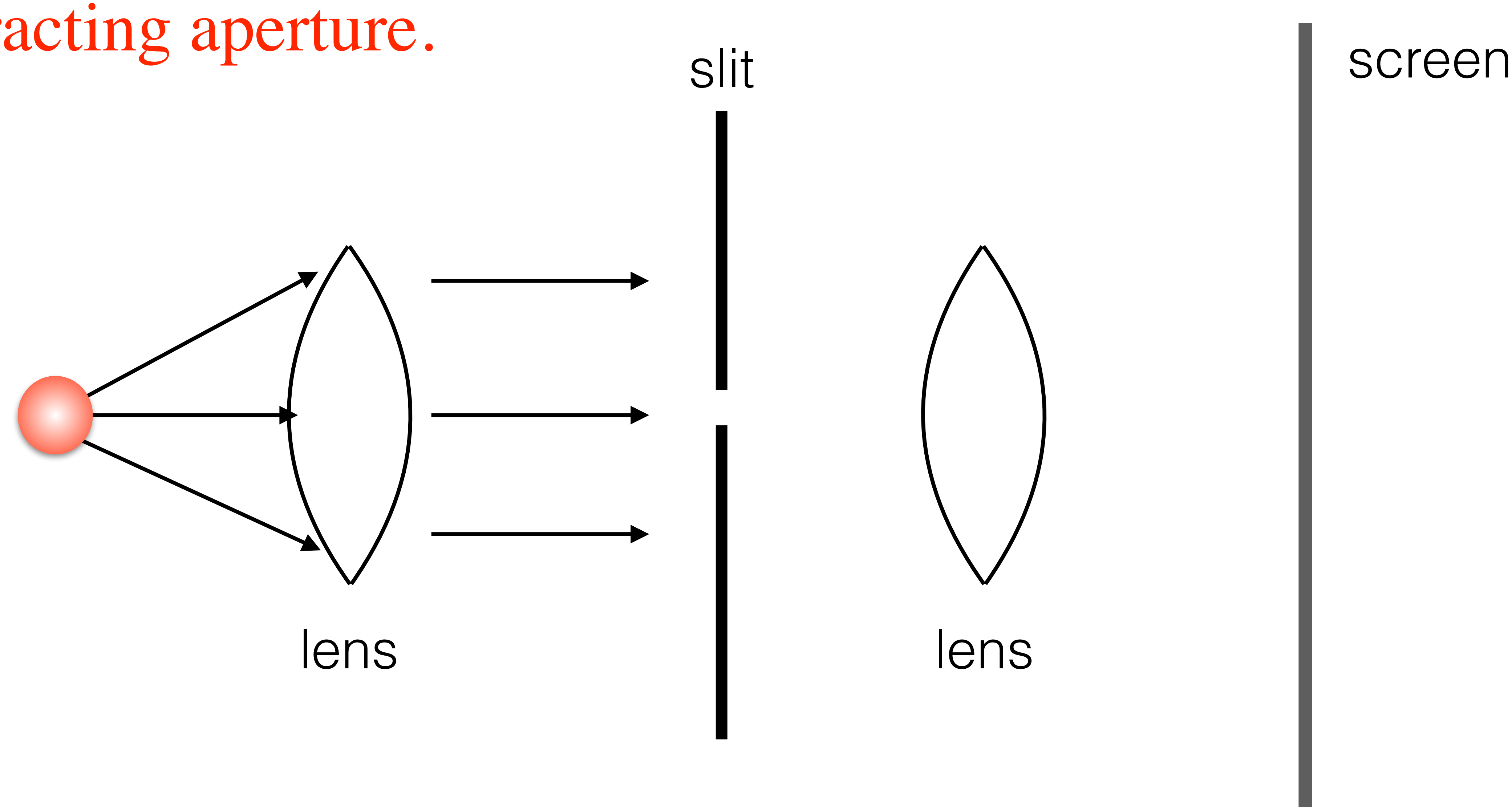


Diffraction

Divided into two categories

Fresnel and Franhofer diffractions

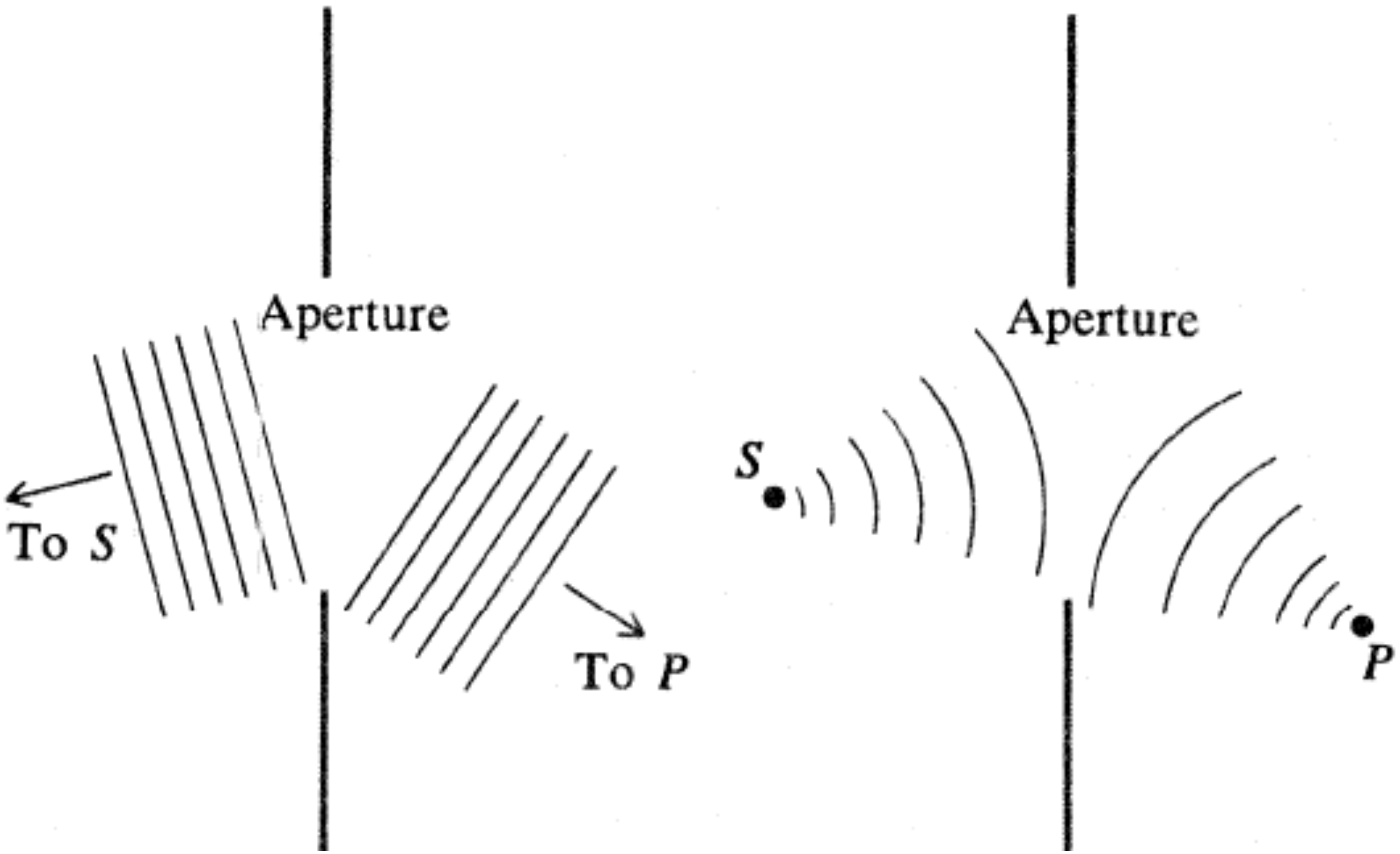
- Franhofer: Both source and screen are at infinite distance from the diffracting aperture.



Diffraction

Divided into two categories

Fresnel and Franhofer diffractions



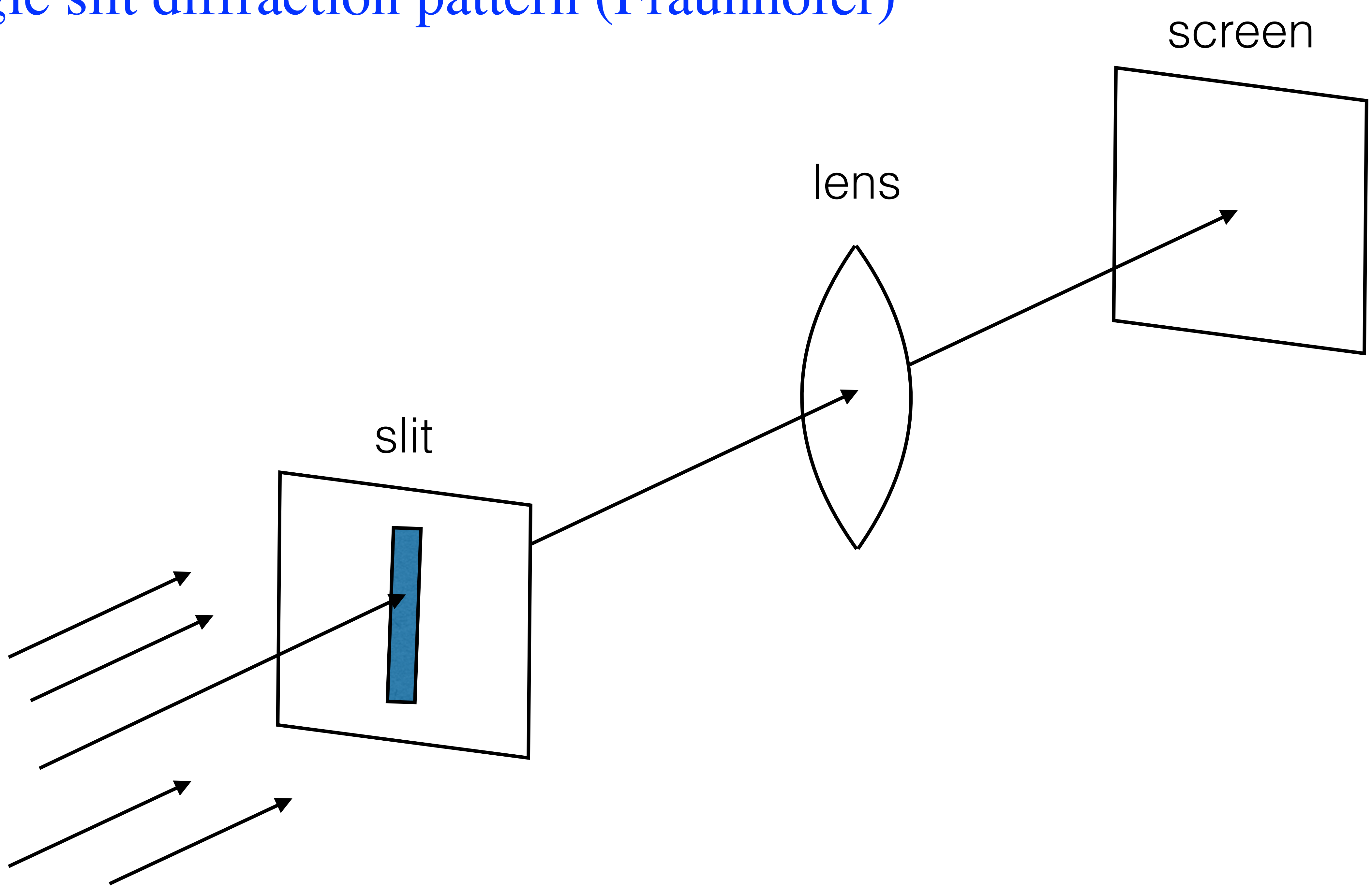
Diffraction

- deviation of light from rectilinear propagation.

Huygen's principle in Wave optics

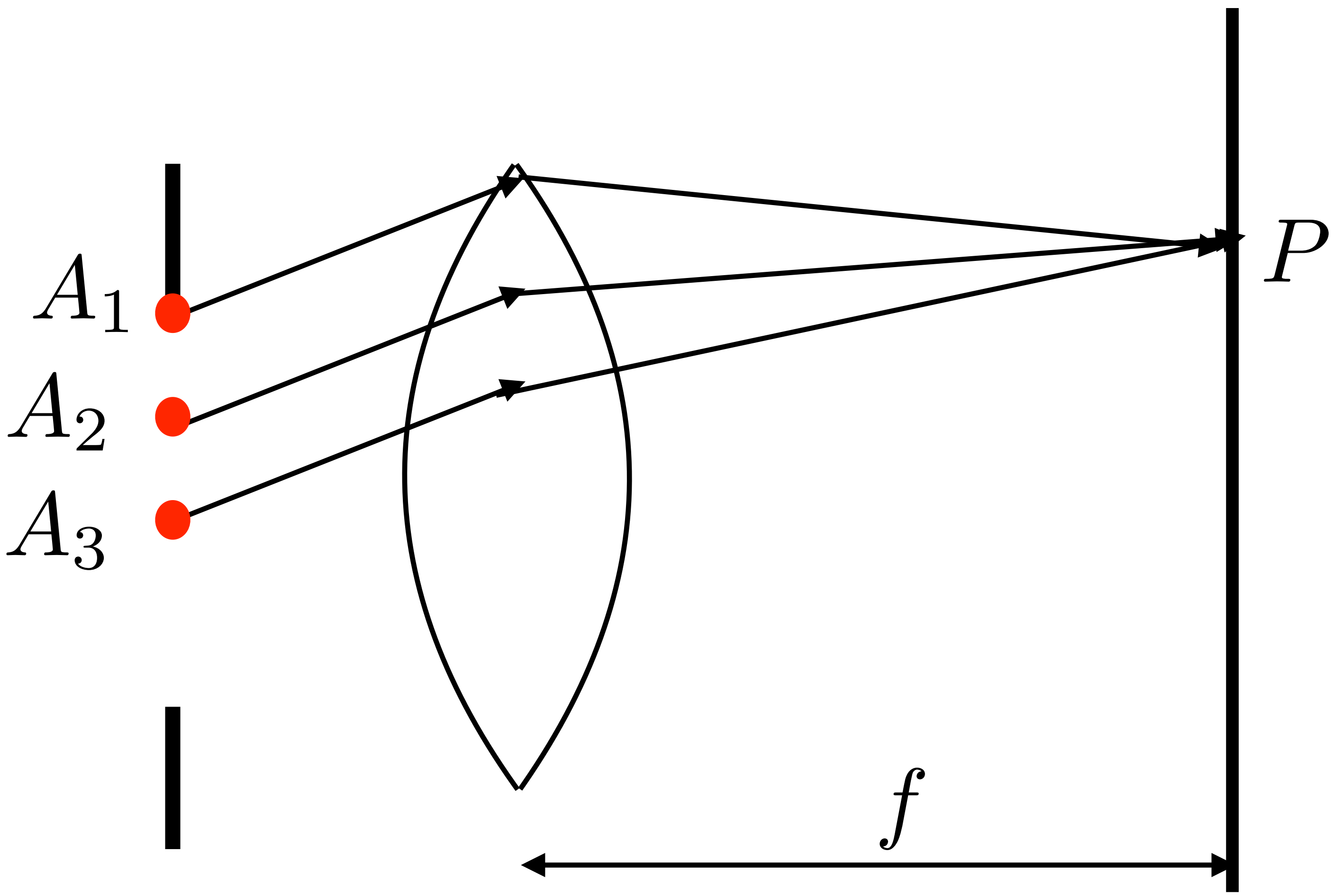
" Every point on a propagating wave front serves as the source of spherical secondary wavelets, such that the wavefront at a later instant of time is the envelope of these wavelets"

Single slit diffraction pattern (Fraunhofer)



- Each point in the slit acts as a source of Huygen's secondary wavelets.
- secondary wavelets interfere to give the patterns.

Single slit diffraction pattern (Fraunhofer)



What is the resultant field at P?

Δ separation between the point sources.

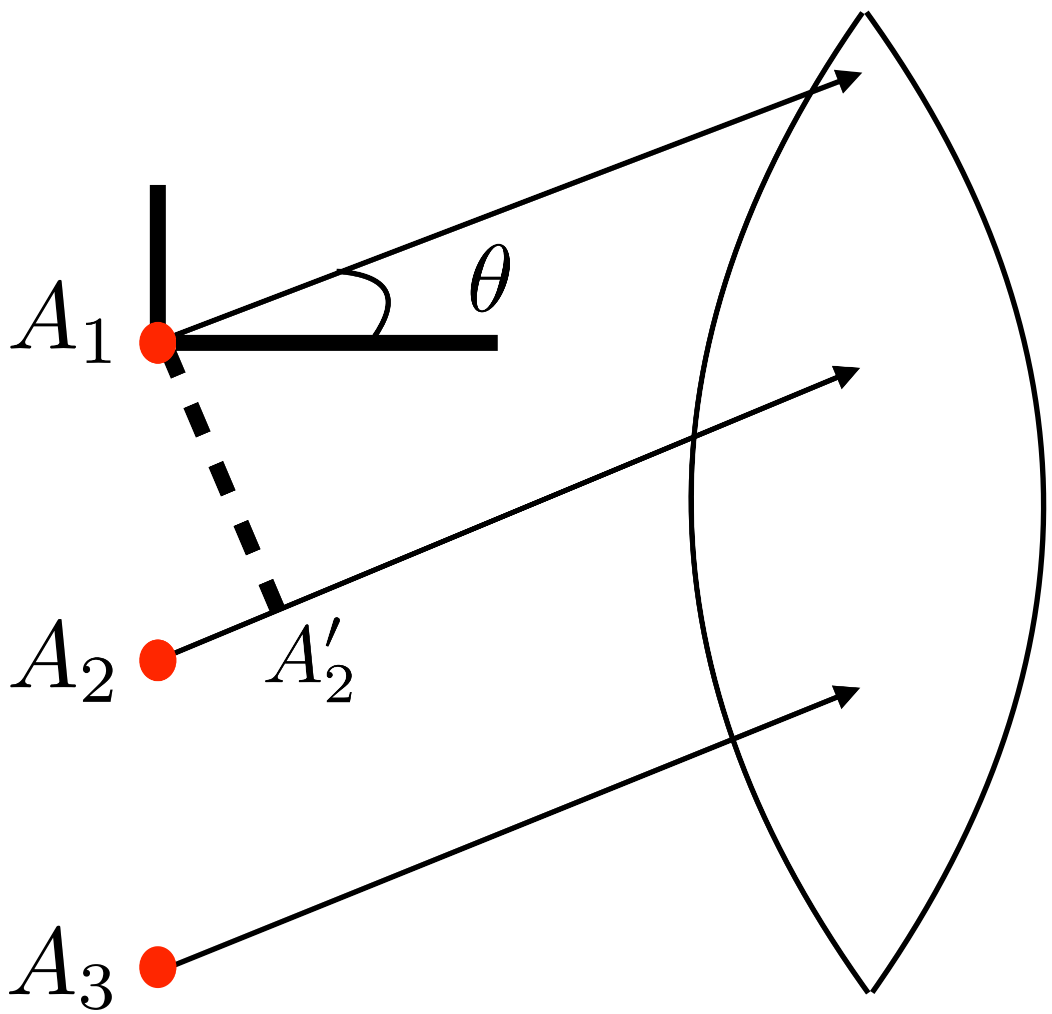
b slit width

n no. of point sources

the fields at P from different sources have different phases due to different path lengths.

$$b = (n-1)\Delta$$

Single slit diffraction pattern (Fraunhofer)



Path difference

$$A_2 A_2' = \Delta \sin\theta$$

Phase difference

$$\phi = \frac{2\pi}{\lambda}\Delta \sin\theta$$

Assuming all sources are equidistant, and have the same amplitudes but differ in the phases, the resultant field is,

$$E = A\left[\cos\omega t + \cos(\omega t - \phi) + ... + \cos(\omega t - (n-1)\phi)\right]$$

Single slit diffraction pattern (Fraunhofer)

$$E = A\left[\cos\omega t + \cos(\omega t - \phi) + \ldots + \cos(\omega t - (n-1)\phi)\right]$$

$$= A\cos\left(\omega t - \frac{1}{2}(n-1)\phi\right)\frac{\sin n\phi/2}{\sin\phi/2}$$ (Home work)

writing as (taking the continuum limit)

$$\phi = \frac{2\pi}{\lambda}\Delta\sin\theta$$

$$E = E_\theta\cos\left(\omega t - \frac{1}{2}(n-1)\phi\right)$$

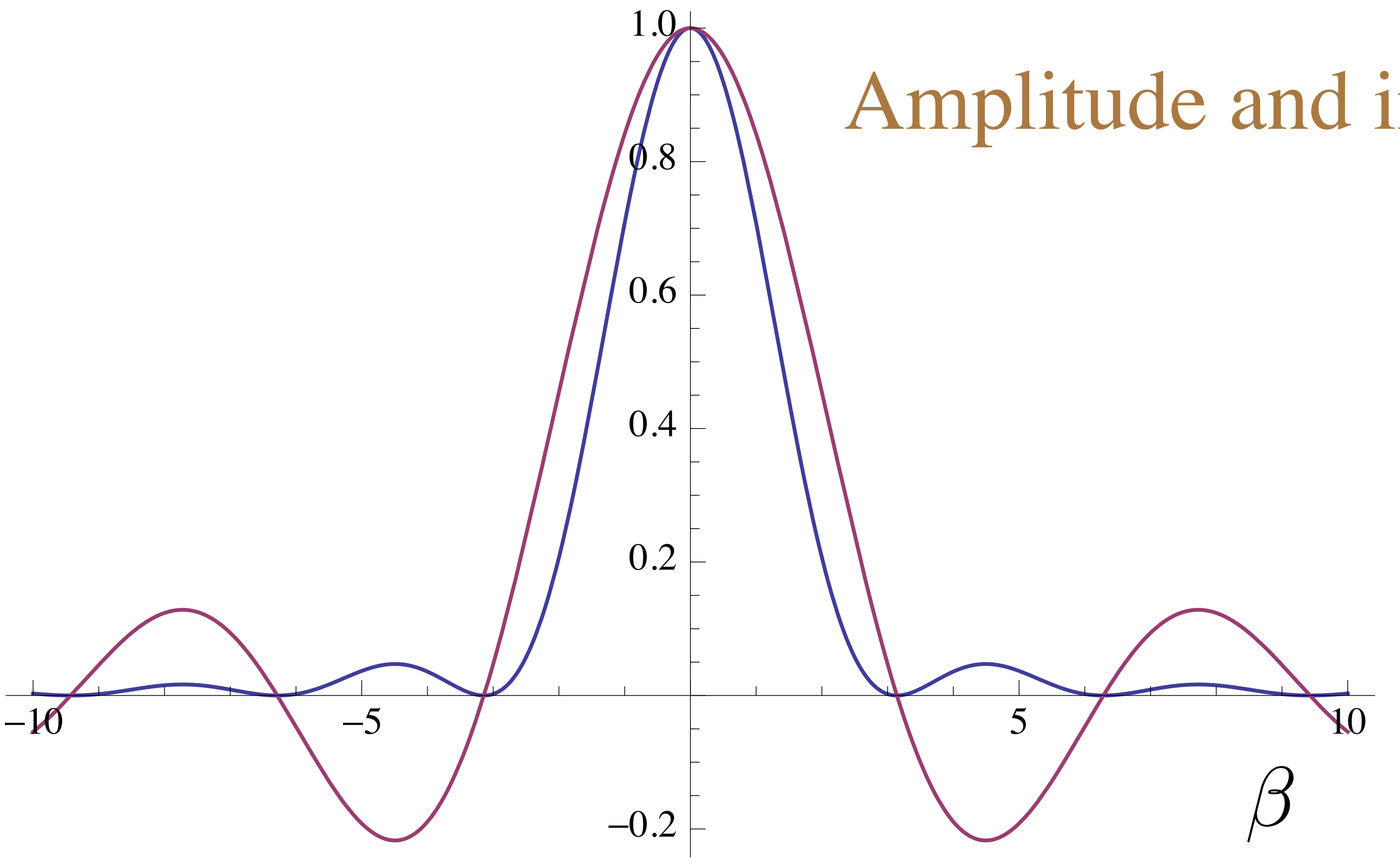
$$E_\theta = nA\frac{\sin\beta}{\beta}$$

$$\beta = \frac{\pi b\sin\theta}{\lambda}$$

$$\boxed{I = I_0\frac{\sin^2\beta}{\beta^2}}$$

I_0 intensity at $\theta = 0$

Single slit diffraction pattern (Fraunhofer)



$$I = I_0 \frac{\sin^2 \beta}{\beta^2}$$

$$\beta = \frac{\pi b \sin\theta}{\lambda}$$

Minima

$$\beta = m\pi, \quad m \neq 0 \;\Rightarrow\; b\sin\theta = m\lambda$$

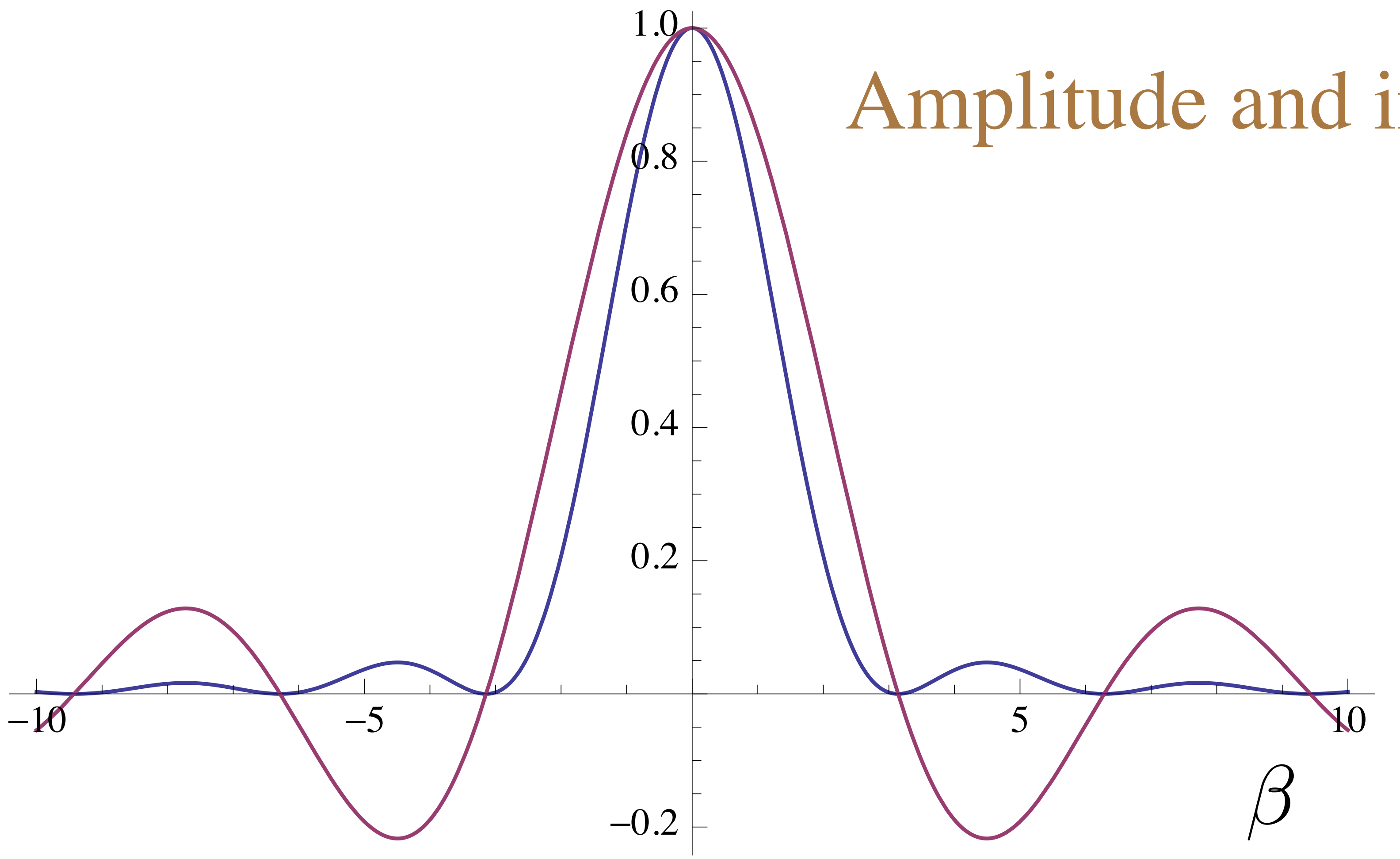
$$m = 0 \;\Rightarrow\; I = I_0$$

angular width of the first peak
(divergence angle)

$$\Delta\theta \sim \lambda/b$$

[Angle over which the most of the energy is concentrated.]

Single slit diffraction pattern (Fraunhofer)



Amplitude and intensity

$$I = I_0 \frac{\sin^2 \beta}{\beta^2}$$

$$\beta = \frac{\pi b \sin \theta}{\lambda}$$

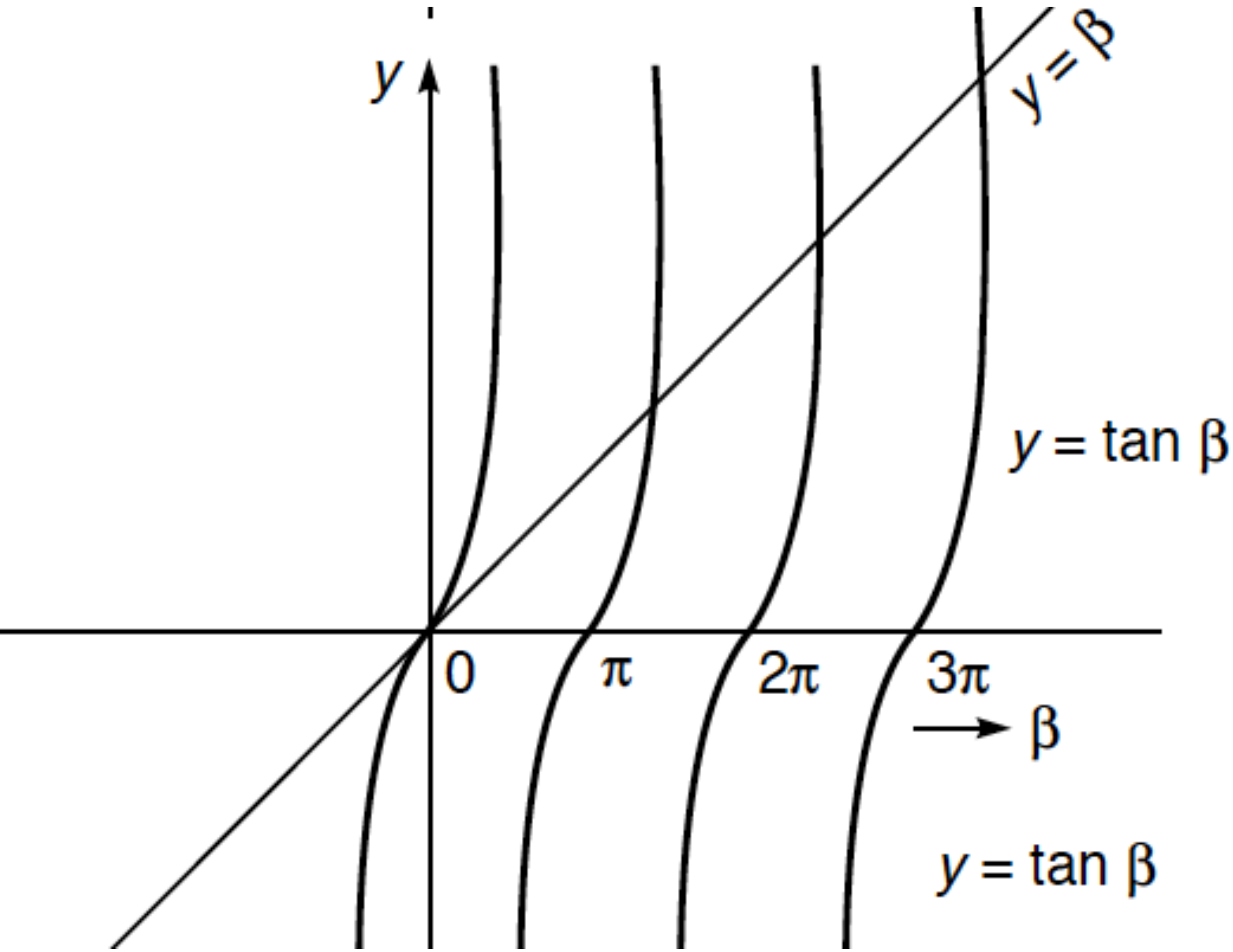
Maxima

$$\frac{dI}{d\beta} = 0 \quad \Rightarrow \quad \sin \beta (\beta - \tan \beta) = 0$$

$$\beta = \tan \beta$$ Transcendental equation

Single slit diffraction pattern (Fraunhofer)

$\beta = \tan\beta$ Transcendental equation



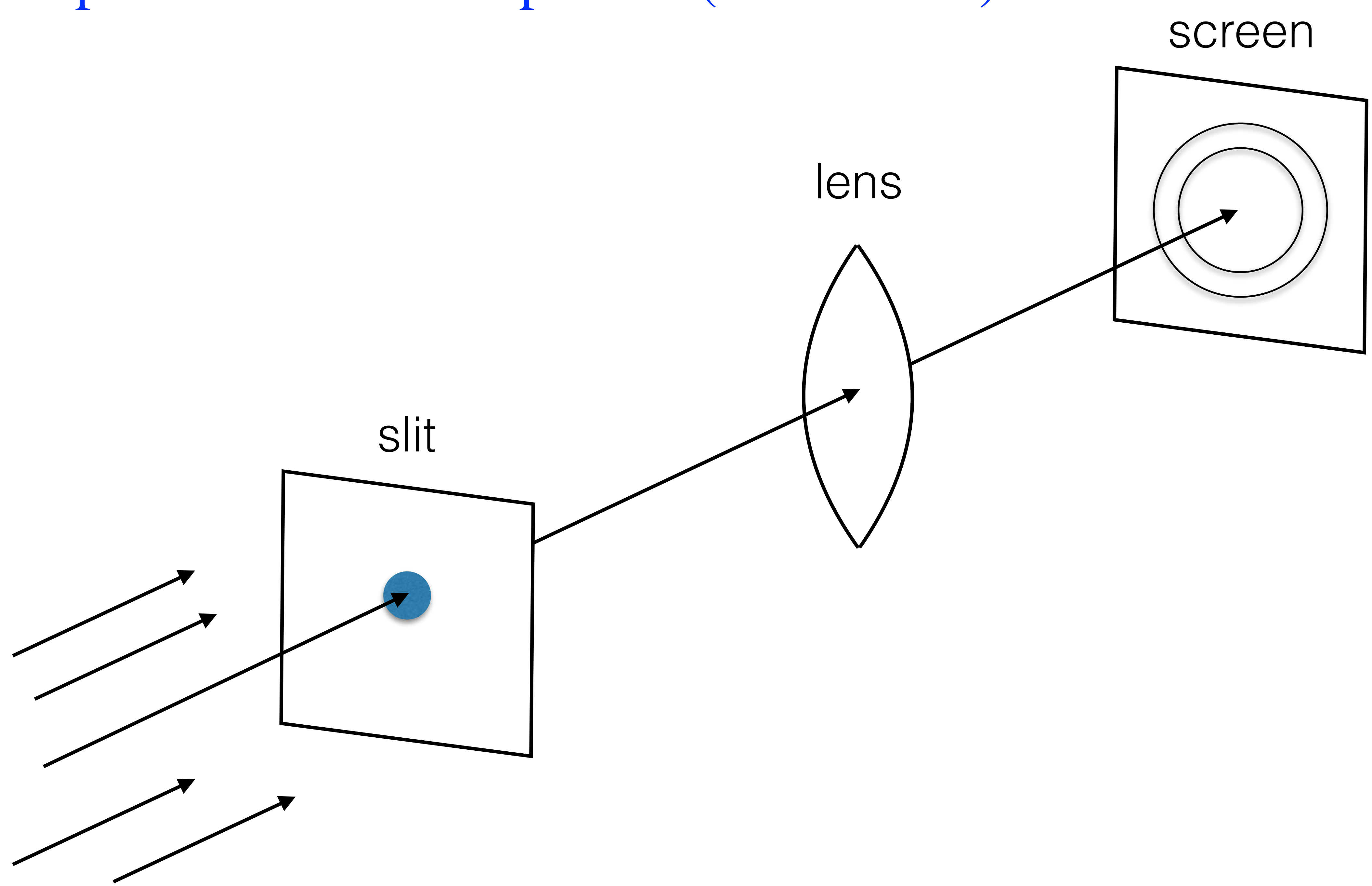
$$I = I_0 \frac{\sin^2\beta}{\beta^2}$$

$$\beta = \frac{\pi b \sin\theta}{\lambda}$$

$$\beta = 1.43\pi,\ 2.46\pi,\ \ldots$$

(Taken from, Optics, Ghatak)

Circular aperture diffraction pattern (Fraunhofer)



Preserving the rotational symmetry leads to *Airy* pattern.

(circular bright and dark fringes)

Circular aperture diffraction pattern (Fraunhofer)

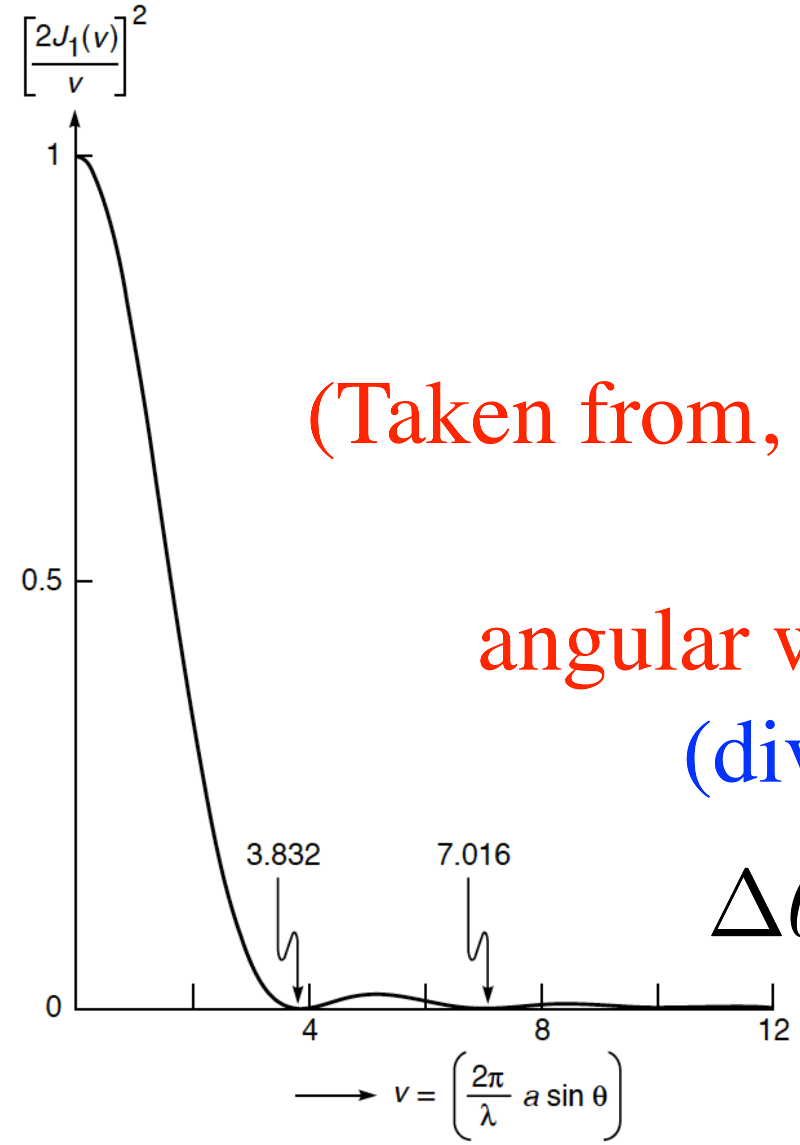
Intensity distribution

$$I = I_0 \left[\frac{2J_1(\alpha)}{\alpha} \right]^2$$

$$\alpha = \frac{2\pi}{\lambda} a \sin\theta$$

Bessel function of the first order.

a is the radius of the circular aperture

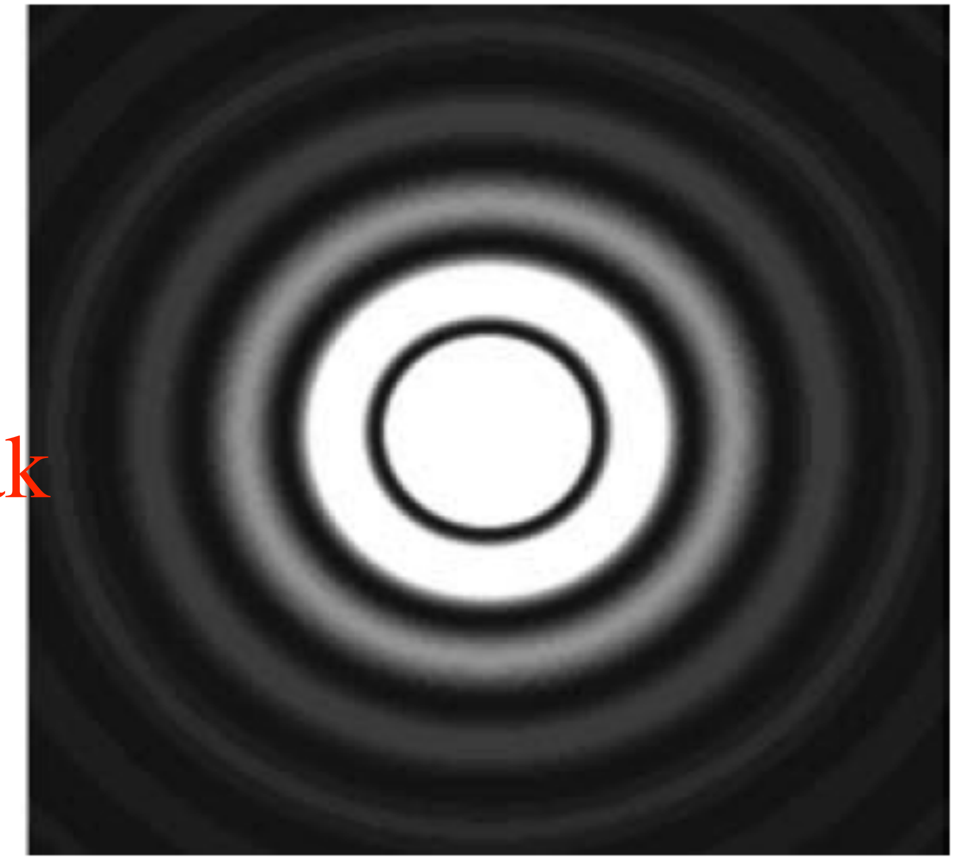


(Taken from, Optics, Ghatak)

angular width of the first peak

(divergence angle)

$$\Delta\theta \approx 0.61\lambda/(2a)$$



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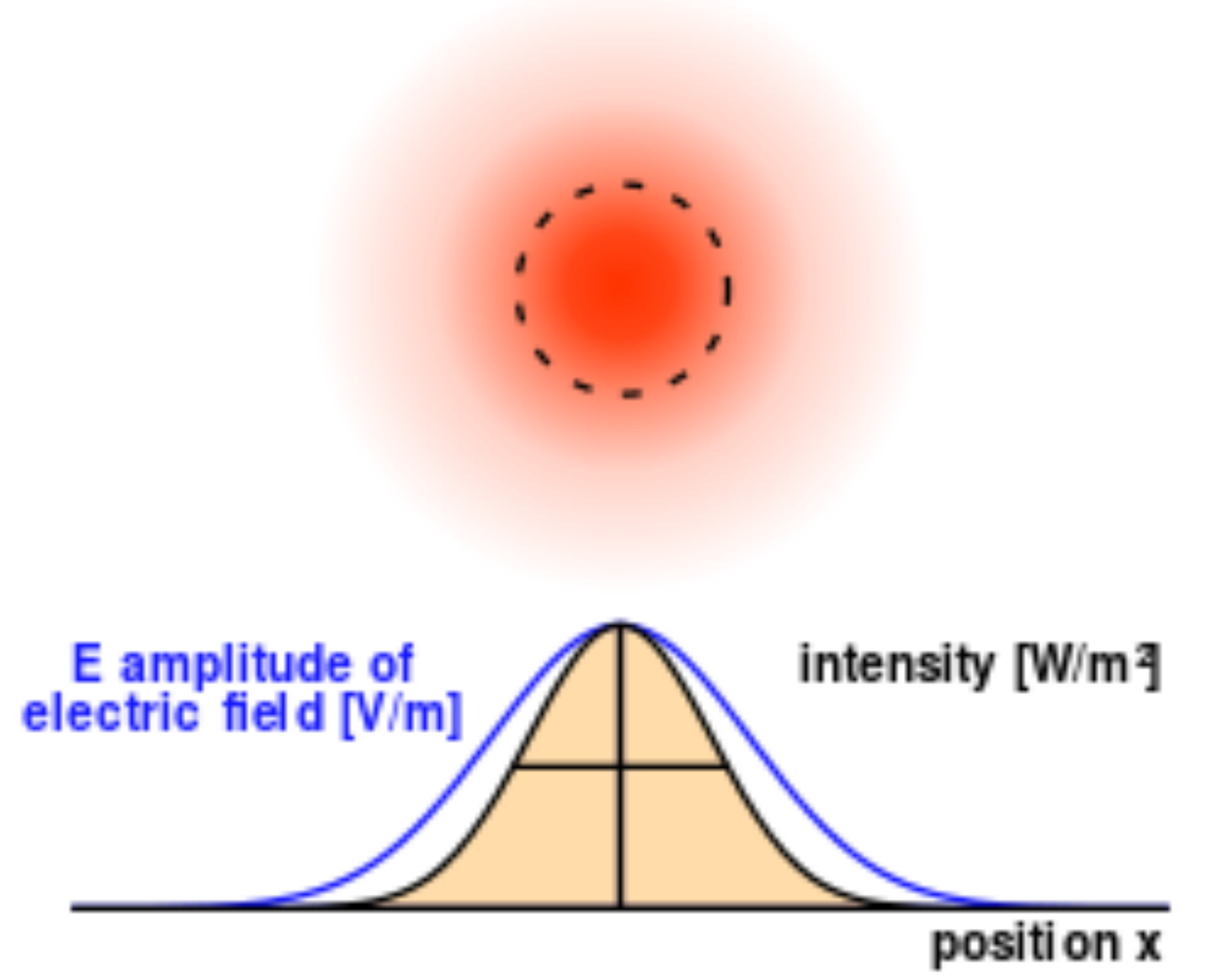
Directionality of laser beams

Ordinary source of light radiates in all directions.

Laser light is highly directional or very small divergence angle.
(It comes from the fact that laser beam comes from a resonant cavity)

Gaussian Beam

Transverse profile (wikipedia: Gaussian beam)



$$E(x,y) = A\exp\left[\frac{-(x^2+y^2)}{w_0^2}\right]$$

spot size of the beam.

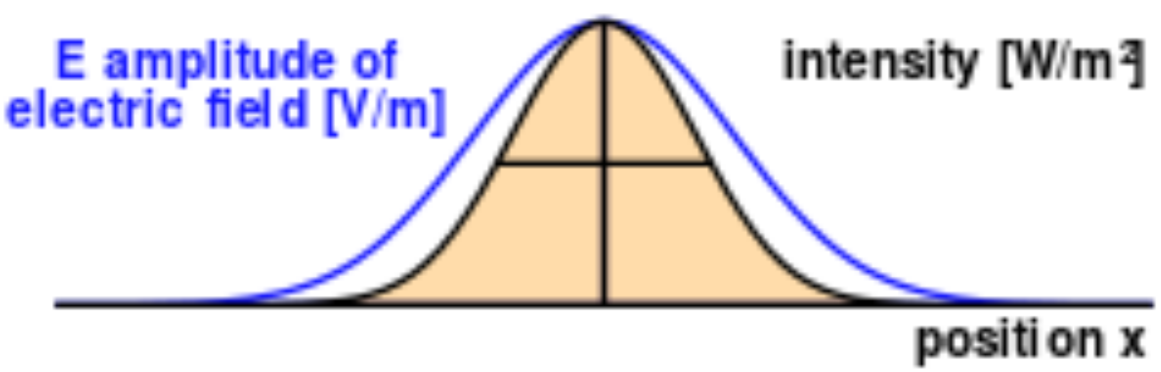
w_0

Directionality of laser beams

Gaussian Beam

Transverse profile (wikipedia: Gaussian beam)

$$E(x,y) = A\exp\left[\frac{-(x^2+y^2)}{w_0^2}\right]$$



the beam propagating along z-direction, hence you can treat z-axis as time axis.

$$I(x,y,z) = \frac{I_0}{1+\gamma^2}\exp\left[-\frac{2(x^2+y^2)}{w^2(z)}\right]$$

Directionality of laser beams

Gaussian Beam

the beam propagating along z-direction, hence you can treat z-axis as time axis.

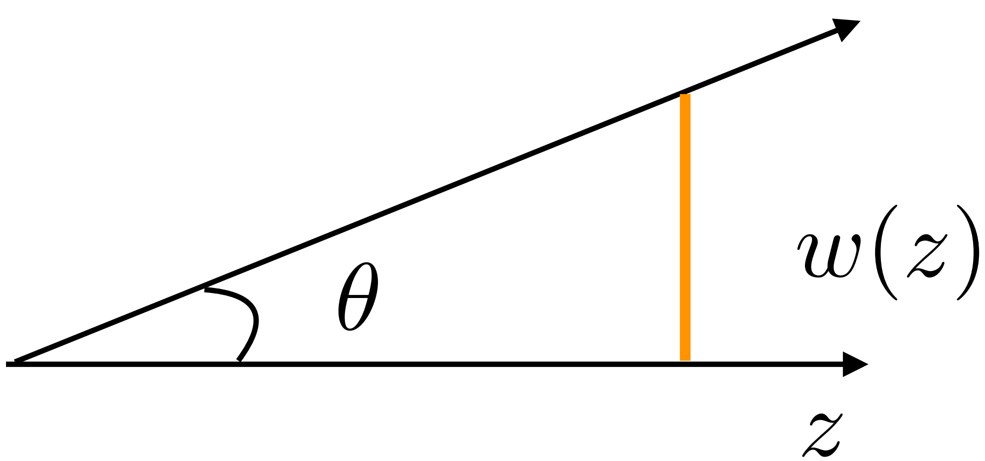
$$w(z) = w_0\sqrt{1+\gamma^2}$$

$$I(x,y,z) = \frac{I_0}{1+\gamma^2}\exp\left[-\frac{2(x^2+y^2)}{w^2(z)}\right]$$

$$\gamma = \frac{\lambda z}{\pi w_0^2}$$

at large z, the width increases linearly with z.

$$w(z) \approx \frac{\lambda z}{\pi w_0}$$ for large z.



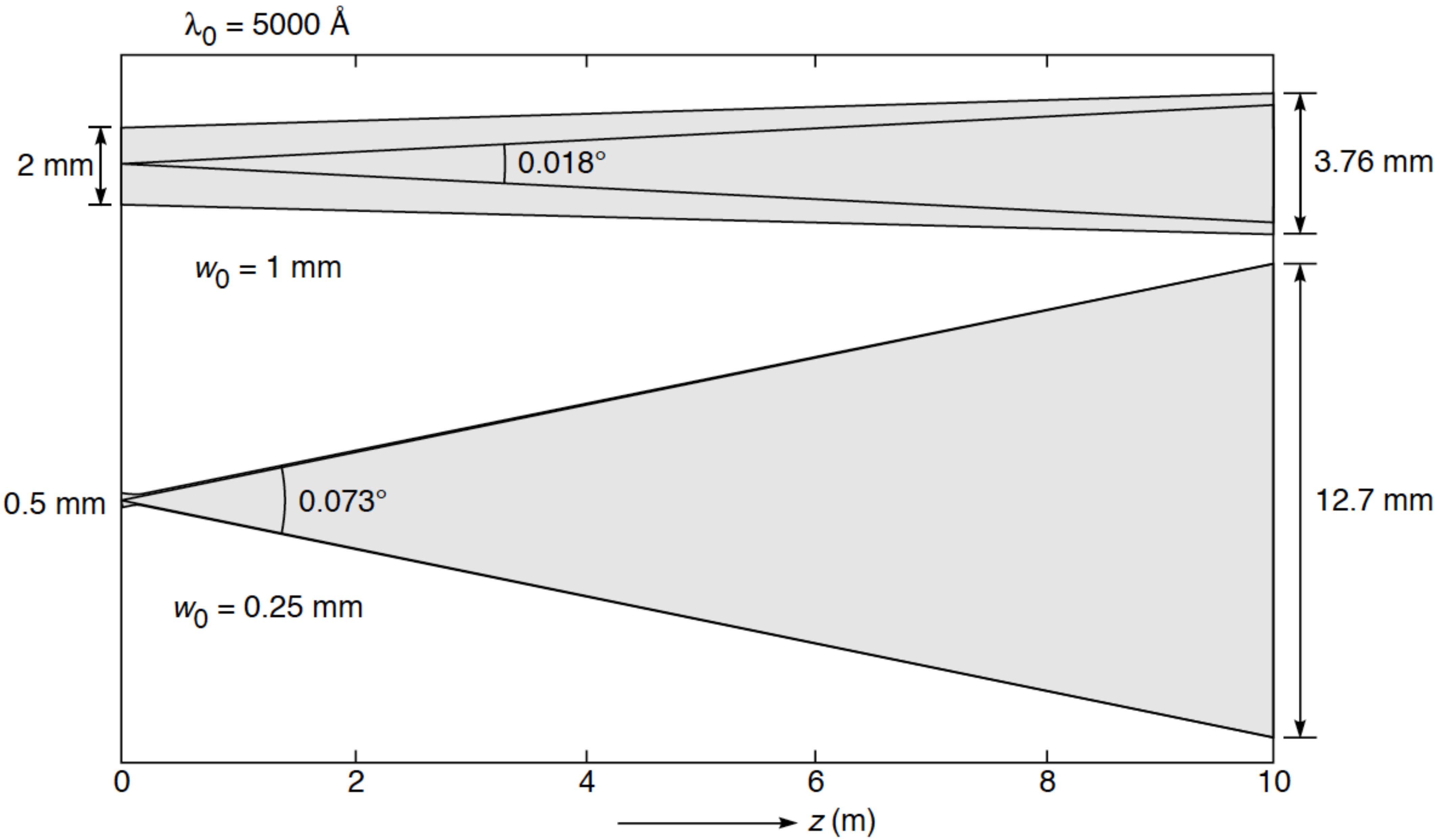
The diffraction angle is defined as

$$\tan\theta = w(z)/z \approx \frac{\lambda}{\pi w_0}$$

Directionality of laser beams

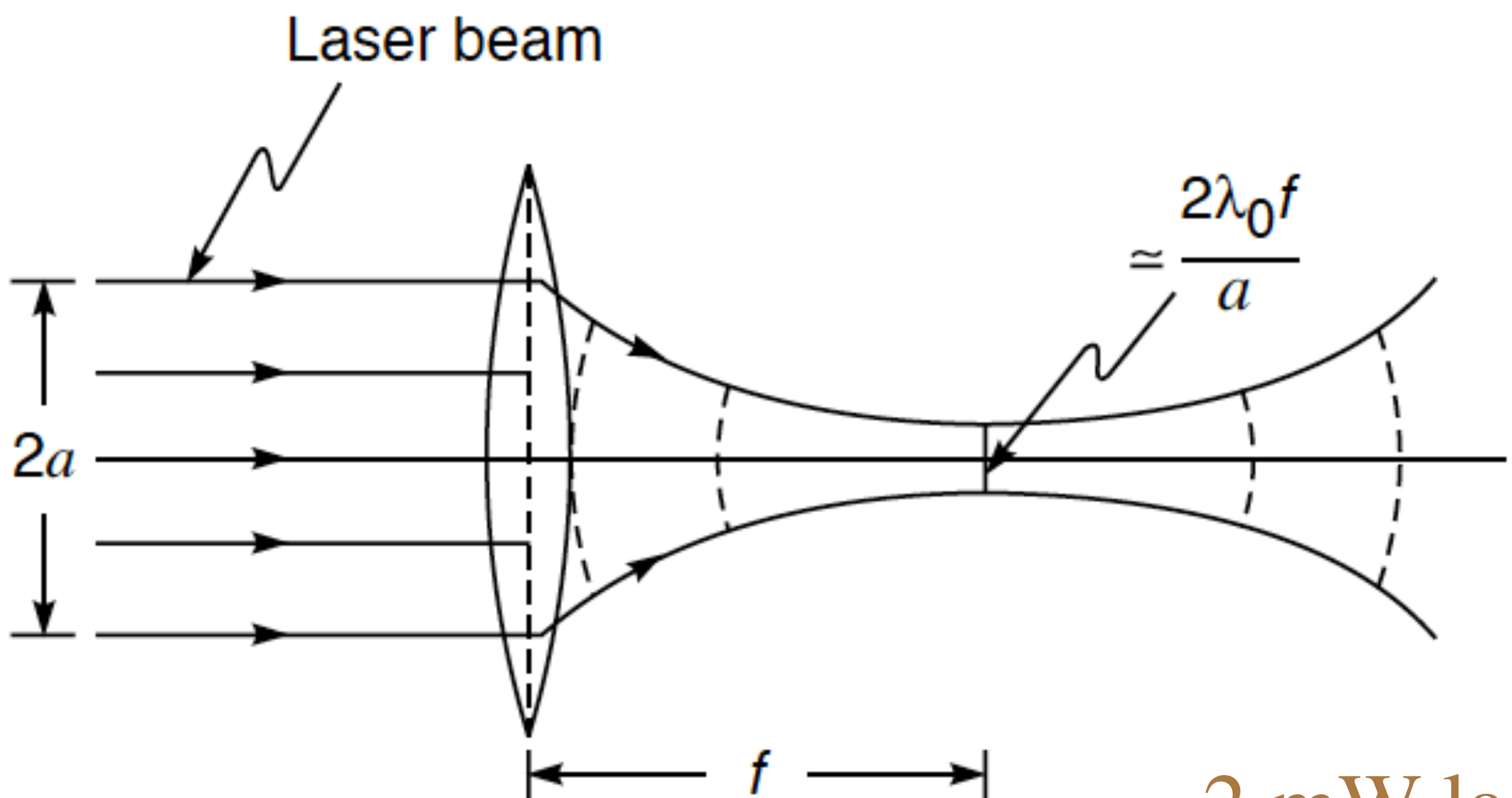
Gaussian Beam

The diffraction angle is defined as $\tan\theta = w(z)/z \approx \dfrac{\lambda}{\pi w_0}$



(Taken from, Optics, Ghatak)

Focusing a laser beam



2 mW laser of wavelength 6×10^{-5} cm, falling on our eye (f=2.5 cm), a=1 mm. What is the intensity on the retina, P/A?

$$3 \times 10^6 W/m^2$$

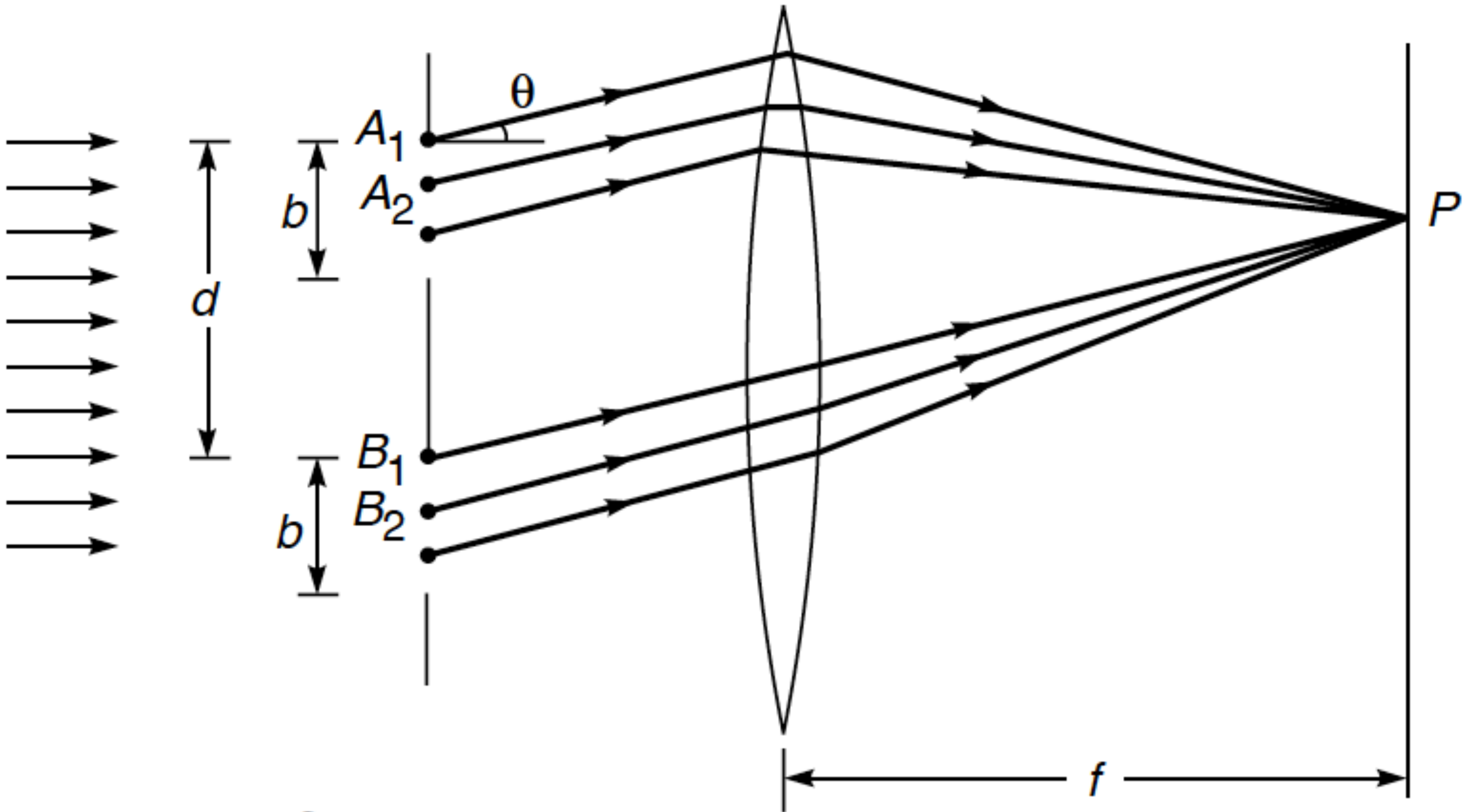
Area of the focused spot,

$$A = \pi \left(\frac{\lambda_0 f}{a} \right)^2$$

(Taken from, Optics, Ghatak)

Two slit diffraction pattern (Fraunhofer)

(Taken from, Optics, Ghatak)



$$E_1 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)$$

$$E_2 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta - \Phi_1)$$

$$\Phi_1 = \frac{2\pi}{\lambda} d\sin\theta$$

phase difference between the two sources.

Two slit diffraction pattern (Fraunhofer)

(Taken from, Optics, Ghatak)

$$E_1 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)$$

$$E_2 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta - \Phi_1)$$

$$\Phi_1 = \frac{2\pi}{\lambda} d\sin\theta$$

phase difference between the two sources.

$$E = E_1 + E_2$$

$$= A\frac{\sin\beta}{\beta}[\cos(\omega t - \beta) + \cos(\omega t - \beta - \Phi_1)]$$

$$\boxed{E = 2A\frac{\sin\beta}{\beta}\cos\gamma\cos\left(\omega t - \beta - \frac{1}{2}\Phi_1\right)}$$

Two slit diffraction pattern (Fraunhofer)

$$E_1 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)$$

$$E_2 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta - \Phi_1)$$

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phase difference between the two sources.

$$E = E_1 + E_2$$

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$$\boxed{E = 2A\frac{\sin\beta}{\beta}\cos\gamma\cos\left(\omega t - \beta - \frac{1}{2}\Phi_1\right)}$$

$$\gamma = \frac{\Phi_1}{2} = \frac{\pi}{\lambda} d\sin\theta$$

Two slit diffraction pattern (Fraunhofer)

$$E = 2A\frac{\sin\beta}{\beta}\cos\gamma\cos\left(\omega t - \beta - \frac{1}{2}\Phi_1\right)$$

$$\gamma = \frac{\Phi_1}{2} = \frac{\pi}{\lambda} d \sin\theta$$

Intensity pattern

single slit pattern

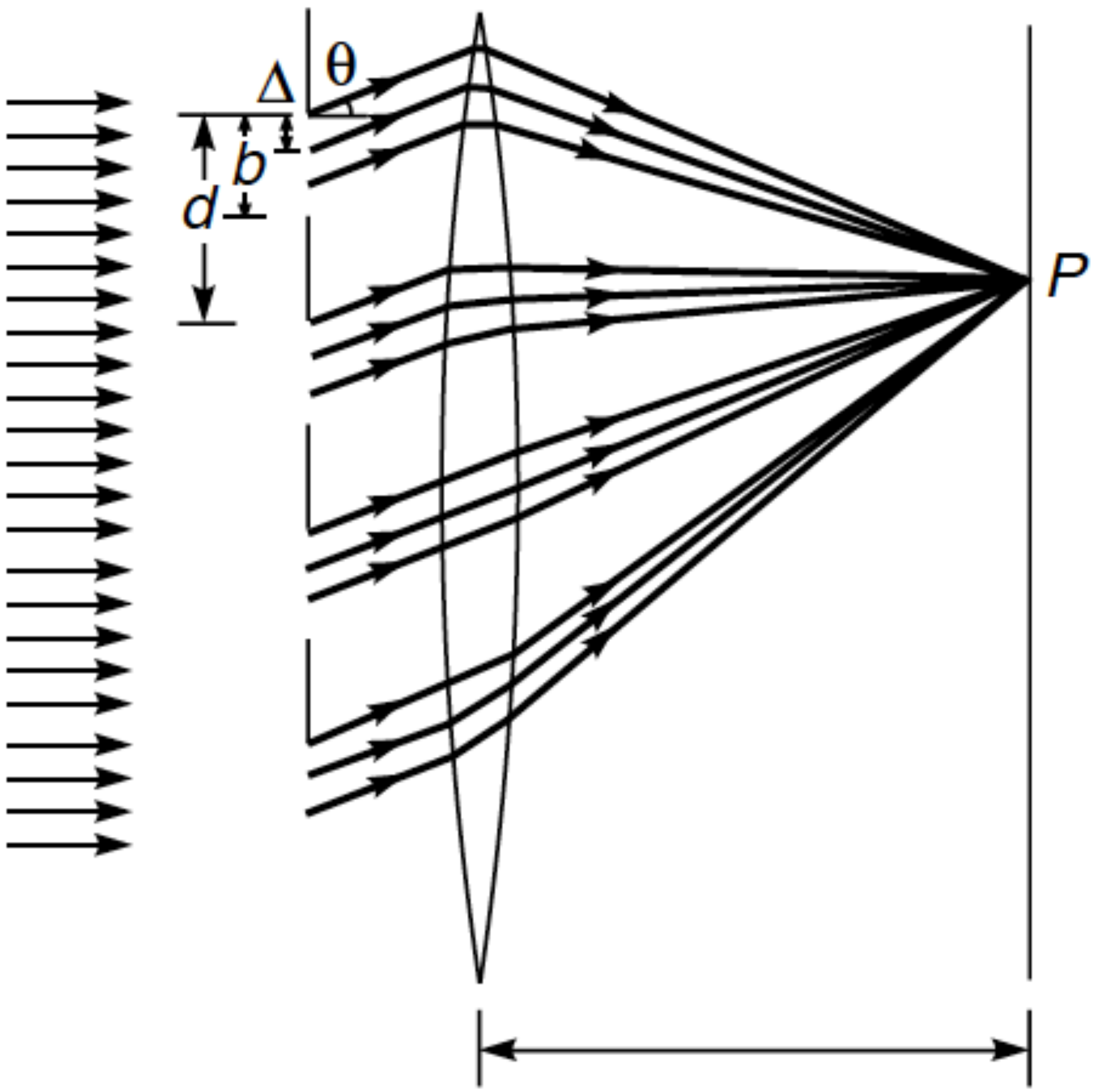
$$I = 4I_0\frac{\sin^2\beta}{\beta^2}\cos^2\gamma$$

$$\cos^2\gamma$$

two point interference pattern

product of the single slit diffraction intensity and interference pattern of two point sources.

N-slits diffraction pattern (Fraunhofer)

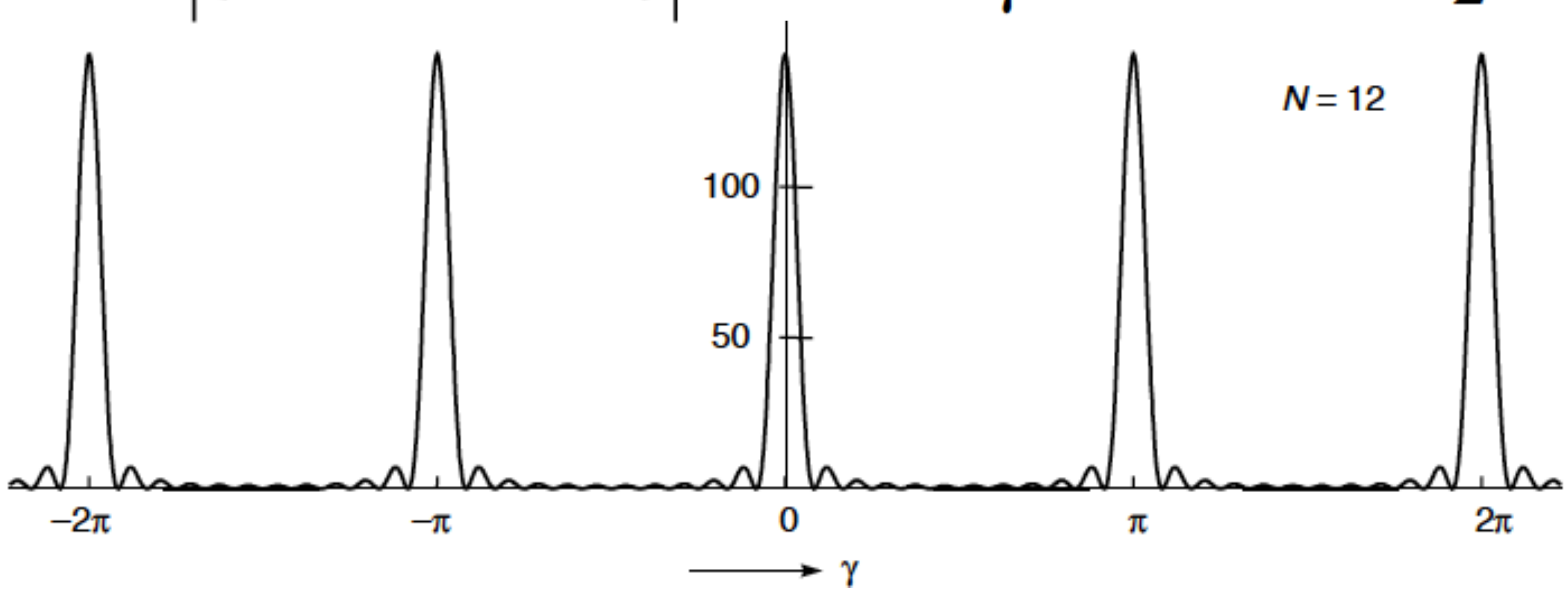


$$I = I_0 \frac{\sin^2\beta}{\beta^2} \frac{\sin^2 N\gamma}{\sin^2\gamma}$$

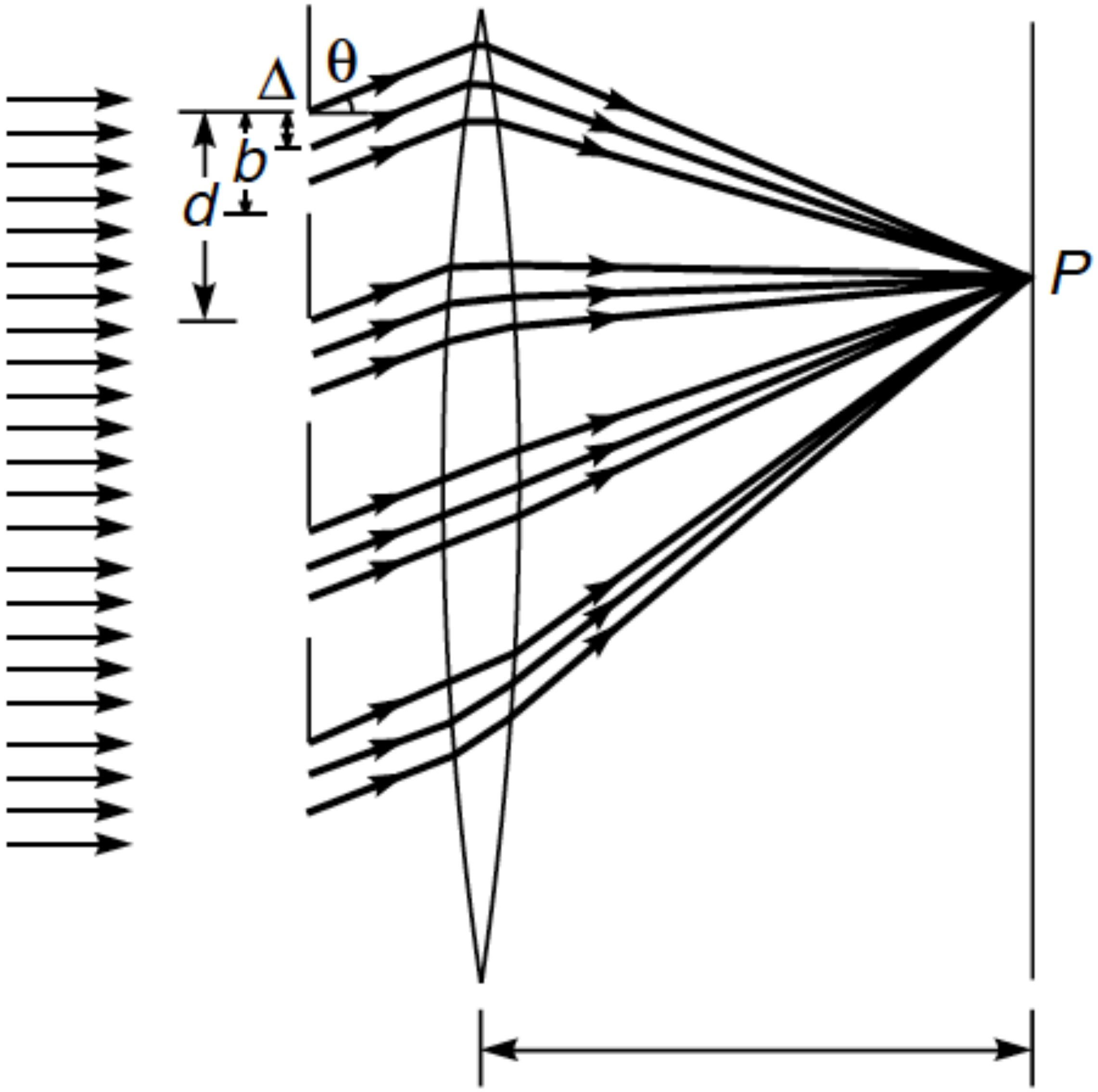
Interference term from N-equally spaced point sources.

$$\frac{\sin^2 N\gamma}{\sin^2\gamma}$$

$$\gamma = \frac{\Phi_1}{2} = \frac{\pi}{\lambda} d \sin\theta$$



N-slits diffraction pattern (Fraunhofer)



$$I = I_0 \frac{\sin^2\beta}{\beta^2} \frac{\sin^2 N\gamma}{\sin^2\gamma}$$

Interference term from N-equally spaced point sources.

$$\frac{\sin^2 N\gamma}{\sin^2 \gamma}$$

$$\gamma = \frac{\Phi_1}{2} = \frac{\pi}{\lambda} d \sin\theta$$

(Home work)

Discuss the maxima and minima of the intensity distribution.

Verify that between two principle maxima there are N-1 minima.

Diffraction grating.