PHY310: Quiz-1

1. Are the following functions analytic ?

a)
$$y^3 - 3x^2y + i(x^3 - 3xy^2 + 2),$$
 b) $e^y(\cos x + i\sin y)$

If analytic, write them as a function of z = x + iy. (3+1)

2. Consider the function

$$f(z) = \frac{z - \sin z}{z^4}.$$

Expand it in a Laurent series around its singularity. Find the first three coefficients in the series. What is the radius of convergence of this series? (4+1)

3. Consider a function f(z) that is analytic in a domain that includes the real axis. When z is real, f(x) is purely imaginary. Show that,

$$f(z^*) = -[f(z)]^*.$$
 (3)

4. Find the squre root of

$$1 - \sqrt{3}i.$$
 (3)