QUIZ-1.

1) a) → satisfies CR cond^{n}s ; hence analytic.

b) → does not satisfy CR cond^{n}s, non analytic.

a) : $y^3 - 3x^2y + i(x^3 - 3xy^2 + 2) = i(z^3 + 2)$.

2) $f(z) = \dfrac{z - \sin z}{z^4}$

$$= \frac{1}{z^4}\left(z - \left(z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!}\right)\right)$$

$$= \frac{1}{z.3!} - \frac{z}{5!} + \frac{z^3}{7!}$$

- First 3 coefficients are $\frac{1}{3!}$, $-\frac{1}{5!}$, $\frac{1}{7!}$
- Radius of convergence → ∞, whole of complex plane except $z=0$.

3) ∵ $f(z)$ is analytic in a domain containing real axis, we can Taylor expand it around the origin,

$$f(z) = a_n z^n \Rightarrow f(z^*) = a_n (z^*)^n.$$

∴ $f(x) = a_n x^n \Rightarrow f(x^*) = a_n (x^*)^n = a_n x^n$.

∵ $f(x)$ is purely imaginary, $[f(x)]^* = -f(x)$.

$\Rightarrow a_n^* x^n = -a_n x^n \Rightarrow \boxed{a_n^* = -a_n}$

Now, $[f(z^*)]^* = a_n^* (z^*)^n = -a_n (z^*)^n = -[f(z^*)]$

proved

4) ~~f(z)=~~

$$1 - \sqrt{3}\,i \equiv 2e^{(-i\pi/3)}.$$

$$\therefore \sqrt{1-\sqrt{3}i} = \pm\sqrt{2}\left(\frac{\sqrt{3}}{2} - i/2\right).$$