ASG-9

1. Prove orthonormality rel^n for Bessel fun^n $J_n(x)$; $n \geq 0$.

2. Using a generating fun^n $g(x,t) = g(u+v,t)$
$= g(u,t) \cdot g(v,t)$
Show that. $J_n(u+v) = \sum_{s=-\infty}^{\infty} J_s(u) \cdot J_{n-s}(v)$.

3. Antenna radiation patterns for a system with circular aperture involves an equ^n,
$$g(u) = \int_0^1 f(r)\, J_0(ur)\, dr.$$
If $f(r) = 1 - r^2$, show that $g(u) = \frac{2}{u^2} J_2(u)$

4. Amplitude of a vibrating circular membrane of radius 'a' satisfies $\nabla^2 u = \frac{1}{v^2} \frac{\partial^2 u}{\partial t^2}$, $u = u(\rho, \phi, t)$
Here, v is the phase velocity. Show that
a) $u(\rho, \phi, t) = J_m(k\rho)\,(a_1 e^{im\phi} + a_2 e^{-im\phi})(b_1 e^{i\omega t} + b_2 e^{-i\omega t})$
b) If $J_m(ka) = 0$, find allowed values of 'k'.