ASG 6 Sol

1). The sol^n around infinity $y(x) \approx \frac{e^x}{\sqrt{2\pi x}}\, p(x)$; putting back in eqn^n, we get an eqn^n for $p(x)$ as.

$$p''(x) + 2p'(x) + \frac{1}{4x^2}\, p(x) = 0. \quad \text{——— (1)}$$

Taking $p(x) = 1 + \frac{b_1}{x} + \frac{b_2}{x^2} + \cdots$ ——— (2). & putting it back in LHS of (1), we get.

$$\frac{1}{x^2}\Big[\underbrace{b_2(25-16x)}_{T_2} + x\big(\underbrace{b_1(9-8x)+x}_{T_1}\big)\Big]. \quad \text{——— (3)}$$

Now, to make T_1 x independent, $-8b_1 x + x = 0 \Rightarrow \boxed{b_1 = \frac{1}{8}}$

Putting $b_1 = \frac{1}{8}$ in (3), we get

$$\frac{1}{x^2}\left[b_2(25-16x) + \frac{9x}{8}\right] \quad \text{——— (4)}$$

To make numerator of (4) x independent, we get, $\boxed{b_2 = \frac{9}{128}}$.

Thus, (2) is an approximate sol^n of (1) to order $\sim \frac{1}{x}$.

2. Sol^n can be obtained by series method and answer is given in Arfken (prob 8.5.5).

To show the series diverges at $x = \pm 1$, using usual ratio test, this series is convergent when.

$$|x| < \lim_{j\to\infty}\left[\frac{a_j}{a_{j+2}}\right] = 1.$$

$\therefore$ at $x = \pm 1$, it diverges.

b) If we choose $n = l$; $l \geq 0$ integer, then ~~and~~ $a_m = 0$ for $m \geq l+2$ [$k = 0$]

[and ~~for~~ ll'ly for $k = 1$].

Thus the series terminates.

c) Can be done by using tricks developed in class.