

ASG - 4 :

1) Evaluate the following Integrals (Results provided).

$$a) \int_0^{\infty} \frac{x^a}{(x+1)^2} dx = \frac{\pi a}{\sin \pi a}, \quad -1 < a < 1.$$

$$b) \int_0^{\infty} \frac{x^{-a}}{x+1} dx = \frac{\pi}{\sin \pi a}, \quad 0 < a < 1.$$

2) Define, $I(y) = \int_{a-iy}^{a+iy} \frac{e^{zt}}{z^{1/2}} dz$. a & t are positive finite.

Show that $\lim_{y \rightarrow \infty} I(y) = 0$.

3). With $f(x) = u(x) + iv(x)$ & $f(x) = f^*(-x)$,
Show that as $x_0 \rightarrow \infty$,

$$u(x_0) \sim -\frac{2}{\pi x_0^2} \int_0^{\infty} x v(x) dx$$

$$\& v(x_0) \sim \frac{2}{\pi x_0} \int_0^{\infty} u(x) dx.$$

4) Show that, in 1D Quantum Mechanics, bound states are non-degenerate.