ASG-2

1. Prove that for real a and b,

$$\left(\frac{ia-1}{ia+1}\right)^{ib} = \exp\left(-2b\cot^{-1}a\right).$$

2. A plane wave of light of angular frequency ω is represented by,

$$e^{i\omega\left(t-\frac{nx}{c}\right)}$$

n = refraction index.

For certain substance, real 'n' is replaced by complex quantity $n - ik$.
What is the effect of k on the wave?

3. A fun^{n} $f(z)$ is analytic. Show that the derivative of $f(z)$ with respect to z^* does not exist unless $f(z)$ is a constant.

4. Show that, $\frac{1}{2\pi i}\oint_C z^{m-n-1}\,dz = \delta_{m,n}$

where 'C' is a closed anticlockwise contour encircling origin.

5. Evaluate $\oint_C \frac{dz}{z^2-1}$, C ≡ circle $|z| = 2$.

6. A fun^{n} $f(z)$ is analytic with in a closed contour C and is continuous on C. If $f(z) \neq 0$ with in C & $|f(z)| \geq M$ on C; show that $|f(z)| \geq M$ for all points with in C.

7. $f(z)$ is a real fun^{n} of z, i.e. $f(z) = f^{*}(z)$. The Laurent expansion of $f(z)$ around origin is $f(z) = \sum a_n z^n$, where $a_n = 0 \ \forall \ n < -N$. Show than a_n are real.

8. Find first three non-zero terms of the Laurent expansion of $f(z) = \dfrac{1}{e^z - 1}$ about the origin.