PHY202: Quiz-1

1. Assuming that black body radiation at absolute temperature T follows Planck's distribution law,

$$\rho(\lambda, T) = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}} - 1},$$

show that maximum wavelength for the distribution is given as,

$$\lambda_{max}T = \frac{hc}{4.965k}.$$

HINT: you may need $\frac{e^{0.035}}{e^5} = 0.00697$. (4)

2. Find the de Broglie wavelength of a mass of 1 g moving at a velocity of $1ms^{-1}$. (2)

3. Consider a wave packet

$$\Psi(x) = Ce^{\frac{i}{\hbar}p_0x}e^{-\frac{|x|}{2\Delta x}},$$

where, C is a constant. Normalising Ψ to unity, find C. (4)

(3) The wave packet is unit normalised.

$$\Rightarrow \int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1.$$

$$\Rightarrow |C|^2 \int_{-\infty}^{\infty} e^{-\frac{|x|}{\Delta x}} = 1.$$

$$\Rightarrow |C|^2 \left[\int_{-\infty}^{0} e^{-\frac{|x|}{\Delta x}} dx + \int_{0}^{\infty} e^{-\frac{|x|}{\Delta x}} dx \right] = 1.$$

$$\Rightarrow |C|^2 \left[\int_{-\infty}^{0} e^{\frac{x}{\Delta x}} dx + \int_{0}^{\infty} e^{-\frac{x}{\Delta x}} dx \right] = 1 \qquad \left[\because |x| = -x,\ x<0;\ |x| = x,\ x>0 \right.$$

$$\Rightarrow |C|^2 \left[\Delta x\, e^{\frac{x}{\Delta x}} \Big|_{-\infty}^{0} + (-\Delta x)\, e^{-\frac{x}{\Delta x}} \Big|_{0}^{\infty} \right] = 1.$$

$$\Rightarrow |C|^2 \left[\Delta x (1-0) - \Delta x (0-1) \right] = 1.$$

$$\Rightarrow |C|^2 \cdot 2\Delta x = 1.$$

$$\therefore |C| = \frac{1}{\sqrt{2\Delta x}} \quad ; \quad \boxed{C = |C| e^{i\alpha}} \quad \alpha = \text{Const}$$

Sol^n of Quiz - 1.

1). $$\rho(\lambda, T) = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda k T}} - 1}$$

~~Max~~^{m} wavelength for the maximum distribution :

$$\frac{\partial \rho}{\partial \lambda} = 0$$

$$\Rightarrow \quad 5\left(e^{\frac{hc}{\lambda k T}} - 1\right) = \frac{hc}{\lambda k T} e^{\frac{hc}{\lambda k T}}$$

$$\Rightarrow \quad x = 5(1 - e^{-x}) \; ; \quad \text{where} \quad x = \frac{hc}{\lambda k T}$$

assume, $x = 5 - a$; a is number.

$$\Rightarrow \quad a = 5 e^{a-5}$$

$$\Rightarrow \quad \frac{a}{5} = \frac{e^a}{e^5}$$

$$\Rightarrow \quad a \simeq 0.035 \quad \Rightarrow \quad x = 4.965$$

$$\therefore \quad \frac{hc}{\lambda_{max} k T} = 4.965$$

$$\Rightarrow \quad \boxed{\lambda_m T = \frac{hc}{4.965 k}}$$

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2) de Broglie wavelength $\lambda = \frac{h}{|\vec{p}|}$.

$$|\vec{p}| = m|\vec{v}| = 1 \times 10^{-3} \text{ kg m/sec}$$

$$h = 6.626 \times 10^{-34} \text{ Jules sec}$$

$$= 6.626 \times 10^{-34} \text{ M}^2 \text{kg / sec}$$

$$\therefore \quad \boxed{\lambda = 6.626 \times 10^{-31} \text{ M}}$$