Assignment 1 [PHY202 (solutions)]

1. The radial component of electric field (E_r) at a distance of r from a charge q is :

$$E_r = \frac{q}{r^2}$$

(stationary charge at t=0)
Now accelerate it(after a time t) by a velocity difference Δv for time Δt So acceleration = a = $\frac{\Delta v}{\Delta t}$ Now we look at the field lines after a time t. Dotted line is E_r's direction for case 1.Now the bold line shows the direction of E_r after the charge has moved.
Let θ = angle between the observer and the charge acceleration vector. Now we can see that there is a tangential component of the electric field(E_t). We want to understand the dynamics for time dt locally at r.

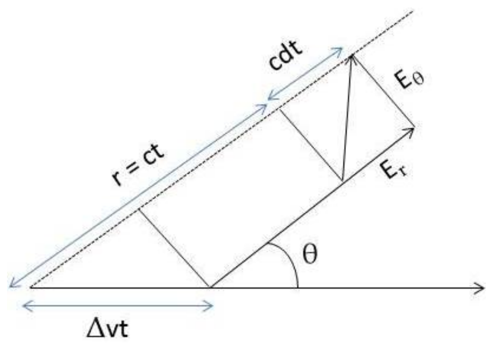

Figure 1: Schematic of Eletric field lines

Ratio of the electric fields in the pulse region:

$$\frac{E_\theta}{E_r} = \frac{\Delta v t \sin\theta}{c\Delta t}$$

$$E_\theta = \frac{q\dot{v}\sin\theta}{rc^2} \qquad \text{(Using r = ct)}$$

$$\implies r^2\dot{r} = -\frac{4}{3}r_0^2 c$$

This is a time varying field, and hence we get electromagnetic radiation. [E_r varies as $\frac{1}{r^2}$ and E_θ varies as $\frac{1}{r}$. So, at large r, E_θ is important.]

By Poynting Law,

$$S = \frac{c}{4\pi}\overline{E} \times \overline{H}$$

where S is the power per unit area

$$\mid S \mid = \frac{c}{4\pi}E^2 \qquad [As \mid E \mid = \mid H \mid]$$

$$\implies \mid S \mid = \frac{c}{4\pi}(\frac{qa\sin\theta}{rc^2})^2$$

The total power radiated is:

$$P = \int_{allspace} \mid S \mid dA$$

$$= \frac{q^2a^2}{4\pi c^3}\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \frac{\sin^2\theta}{r^2} r\sin\theta$$

$$\implies P = \frac{2}{3}\frac{q^2a^2}{c^3}$$

2. We will use the Larmor formula to find the energy loss.

$$\frac{dU}{dt} = -P = -\frac{2e^2a^2}{3c^3}$$

In the non-relativistic limit,

$$U = -\frac{e^2}{r} + \frac{1}{2}m_0v^2$$

Also using the classical formula for Coulomb force:

$$m_0a = \frac{e^2}{r^2} = m_0\frac{v^2}{r}$$

$$U = -\frac{e^2}{2r}$$

$$\frac{dU}{dt} = \frac{\dot{r}e^2}{2r^2}$$

We finally get:

$$\frac{\dot{r}e^2}{2r^2} = -\frac{2}{3}\frac{e^2}{c^3}\frac{e^4}{m_0 2r^4}$$

$$\implies r^2\dot{r} = -\frac{4}{3\frac{e^4}{c^3m_0^2}}$$

$$\frac{1}{3}\frac{d}{dt}(r^3) = -\frac{4}{3}\frac{e^4}{c^2m_0^2}t$$

$$\implies r^3 = const - 4\frac{e^4}{c^2m_0^2}$$

At t = 0, r = radius of the atom = a_0 Therefore, $r^3 = a_0^3 - 4\frac{e^4}{c^2 m_0^2} t$
When the electron falls to the centre, r = 0 So, the time for the fall will be:
$t = \frac{a_0^3 c^2 m_0^2}{4e^4} \approx 1.6 \times 10^{-11} s.$

3. If a $\ll L$, the difference in path length travlled by two rays from A and B to O is: BP $= r_1 - r_2 = a \sin\theta$
If the rays were in phase when they pass through the slits, then the condition for constructive intereference is:

$$d \sin\theta = m\lambda, \qquad m = 0, \pm 1, \pm 2, ...$$

And the condition for destructive intereference is:

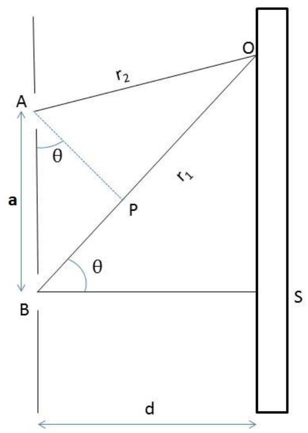

Figure 2: Young's Double slit experiment

$$d \sin\theta = (m + \frac{1}{2})\frac{\lambda}{2}, \qquad m = \pm 1, \pm 2, ...$$

The region of constructive interference gives the bright spot while the region of destructive interference gives the dark spot. The dark and bright spots are the interference fringes.

$$\sin\theta = \frac{OS}{d} ...$$

For m^{th} bright spot,

$$OS^B = \frac{\lambda m d}{a}$$

For m^{th} dark spot,

$$OS^B = \frac{\lambda(m+\frac{1}{2})d}{a}$$

So the space between two consecutive fringes $= \dfrac{\lambda d}{a}$

4. The time average of $f(t) = \cos^2(\omega t - x)$ is evaluated as follows:

$$\frac{\int_0^{2\pi} \cos^2(\omega t - x)dt}{\int_0^{2\pi} dt}$$

$$\begin{aligned}
\text{Now} \int_0^{2\pi} \cos^2(\omega t - x)dt &= \frac{1}{2}\int_0^{2\pi} dt \ + \ \frac{1}{2}\int_0^{2\pi} \cos 2(\omega t - x)dt \\
&= \pi + \frac{1}{4}\left[\frac{\sin(\omega t - x)}{\omega}\right]_0^{2\pi} \\
&= \pi + \frac{1}{4\omega}[\sin(2\pi\omega - x) - \sin(-x)] \\
&= \pi + \frac{1}{4\omega}[-\sin(x) - \sin(x)] \quad \text{as } \omega = 1 \\
&= \pi
\end{aligned}$$

$$\text{And} \int_0^{2\pi} dt = 2\pi$$

So the time average is $\frac{\pi}{2\pi} = \frac{1}{2}$

The time average of $f(t) = \cos(\omega t - x)$ is evaluated as follows:

$$\frac{\int_0^{2\pi} \cos(\omega t - x)dt}{\int_0^{2\pi} dt}$$

$$\begin{aligned}
\text{Now} \int_0^{2\pi} \cos(\omega t - x)dt &= \frac{1}{\omega}\left[\frac{\sin(\omega t - x)}{\omega}\right]_0^{2\pi} \\
&= \frac{1}{\omega}[\sin(2\pi\omega - x) + \sin(x)] \\
&= \frac{1}{\omega}[\sin(2\pi - x) + \sin(x)] \\
&= \frac{1}{\omega}[\sin(-x) + \sin(x)] \\
&= 0
\end{aligned}$$

So the time average is $\frac{0}{2\pi} = 0$