

Topology

Semester I

- Smooth structures, Smooth maps, Bump functions
- Partitions of Unity, Extension Lemma, Tangent Vectors, Pushforwards
- Tangent Bundles, Vector fields, Vector Bundles, Bundle Maps, Sections, Cotangent Bundles
- Inverse/Implicit Function Theorem, Submanifolds, Submersions, Level sets, Statement of Sard's Theorem
- Differential Forms, Wedge product, Exterior Derivative, Definition only of De Rham Cohomology groups
- Orientation, Riemannian Volume form
- Integration on Manifolds using differential forms, Stoke's Theorem
- Fundamental groups and its properties, Fundamental group of the circle, Brouwer's fixed point for 2-disks.
- Van Kampen Theorem, Fundamental group of CW-complexes
- Covering spaces, Lifting properties
- Classification of covering spaces, deck transformations
- Higher homotopy groups, Commutativity of higher homotopy groups, Higher homotopy groups of covers

Semester II

- Delta Complexes, Simplicial and Singular Homology, Relation between Homology and Fundamental group
- Homotopy Invariance, Relative Homology, Exact sequences, Brouwer's fixed point for n-disks
- Excision, Mayer-Vietoris, Statement of Equivalence of simplicial and singular homology
- CW-complexes, Degree, Cellular homology, Euler Characteristic
- Homology with coefficients, Axioms for Homology, Statement of Lefschetz fixed point theorem, Statement of Invariance of domain,
- Orientation of topological manifolds, Orientation in terms of Homology
- Idea of Cohomology, Statement of Poincare Duality
- Universal Coefficients Theorem
- Cohomology of spaces, Cross and Cup products
- Cohomology Ring, Statement of the basic Kunneth Formula,
- Computation of Cohomology rings, Statement of DeRham comparison theorem

References

- A. Hatcher, "Algebraic Topology"
- J. Lee, "Introduction to Smooth Manifolds"
- G. Bredon, "Topology and Geometry"
- Greenberg and Harper, "Algebraic Topology: A first course"
- Guilleman and Pollack, "Differential Topology"