Probability theory

Semester 1

- The Probability Space, Events, properties of probability measures, independence, Bayes' formula, Kolmogorov 0-1 law
- Random variable, distribution functions, examples of discrete and continuous distributions, joint distributions, independence of random variables, Borel-Cantelli lemmas
- Limits theorems: Weak/Strong Law of Large numbers, Central limit theorem
- Conditional probability, Martingales, Stopping time, Azuma's inequlity, Doob's inequalities.
- Discrete time discrete space Markov chain, Chapman-Kolmogorov equation, classification of states, and limit theorems.

Semester 2

- Measure theoretic probability, Kolmogorov consistency theorem, notion of convergence, Monotone class theorem, weak convergence, Portmanteau theorem, Skorohod representation theorem, tightness and compactness, characteristic functions and its properties.
- Conditional expectation and Jensen's inequality, regular conditional expectation. Markov processes.
- Definition of Brownian motion, Lévy's Theorem, construction of Brownian motion, Some properties of Brownian motion, Geometric Brownian motion.
- Stochastic calculus, Ito integration, Ito's formula, stochastic differential equations, strong/weak solutions.

References

- Allan Gut - Probability: A Graduate Course (Springer Texts in Statistics)
- Rick Durrett - Probability: Theory and Examples (Cambridge Series in Statistical and Probabilistic Mathematics)
- K. B. Athreya and S.N. Lahiri: Measure Theory and Probability Theory , (Springer Texts in Statistics).
- Peter Mörters and Yuval Peres: Brownian motion. Cambridge Series in Statistical and Probabilistic Mathematics)
- Ioannis Karatzas and Steven E. Shreve: Brownian motion and stochastic calculus.
- Bernt Oksendal. Stochastic differential equations, An introduction with applications. Universitext.