Indian Institute of Science Education and Research Pune

Comprehensive Exams – 2016

Subject: Topology
Date: June 10, 2017

Time duration: 3 hours
Total Points: 60

Instructions

- This exam has 6 questions, for a total of 60 points.
- All questions are mandatory.
- Please show all your work clearly and legibly.
- Quote any theorem or result you use.

Q.1) Let ω be the one form

$$\omega = (x^2 + y)\,\mathrm{d}x + (y^3 + x)\,\mathrm{d}y$$

on the unit disk $D = \left\{(x,y) \in \mathbb{R}^2 \,\middle|\, x^2 + y^2 \le 1\right\}$. Compute

$$\int_{S^1} \omega$$

where $S^1 = \left\{(x,y) \in \mathbb{R}^2 \,\middle|\, x^2 + y^2 = 1\right\} \subset D$. [10]

Q.2) Let S be a compact surface (without boundary) of genus 2.

(a) Compute the fundamental group $\pi_1(S)$. [4]

(b) Compute the homotopy group $\pi_2(S)$. [6]

Q.3) Compute the homology groups $H_n(M, M \smallsetminus \{x\})$ where M is an n-dimensional manifold and $x \in M$ is a point. [10]

Q.4) Suppose M is a connected, compact 5-dimensional manifold without boundary such that $\pi_1(M) \cong \mathbb{Z}/4\mathbb{Z}$.

(a) What is $H^1(M, \mathbb{Z}/2\mathbb{Z})$? [6]

(b) What is $H^5(M, \mathbb{Z}/2\mathbb{Z})$? [4]

Q.5)

(a) Compute the cohomology ring $H^*(T^2, \mathbb{Z})$. State the results used clearly. [7]

(b) What are the de Rham cohomology groups of T^2? [3]

Q.6)

(a) Describe $\mathbb{CP}^3$ as a cell complex. [4]

(b) Compute the cellular homology groups of $\mathbb{CP}^3$. [6]

End of exam