<u>Algebraic Number Theory</u>

1. Algebraic numbers and algebraic integers: definitions and basic propertiesChap I: Section I and II Neukirch

2. Dedekind domains, prime ideals, ideal factorization, Examples Chapt I : section 8 Neukirch

3. Quadratic reciprocity Chap I : Section 8 Neukirch

4. Finiteness of class group Chap 6 : Esmonde, Murty

5. Ramification index, inertial degree, Decomposition groups and inertia groups, ExamplesChap I: section 9 Neukirch

6. Local fields: algebraic, analytic and topological definition and their equivalence, valuation, absolute value, completions, examples, topological properties, ring of integers is a local ring.Chap II: First 5 sections Neukirch

7. Dirichlet's Unit Theorem: Statement and idea of the proof.

Chap 8: Esmonde and Murty

8. Infinite Galois theory: Krull topology.

References:

1. Neukirch : Algebraic Number Theory.
2. Esmonde and Murty: Problems in Algebraic number theory.

Algebraic Geometry

Topics:

1. Hilbert Basis Theorem. Noether normalization, Nullstellensatz.
2. Affine and Projective varieties.
3. Morphisms of Varieties: Veronese and Segre embeddings.
4. Rational maps. Birationality.
5. Singularities and non-singular varieties.
6. Blowing up at points. Normalization. Desingularization.
7. Algebraic curves. Field of rational functions.
8. Sheaves and definition of schemes.
9. Basic properties of schemes: integral schemes, projective schemes. (Separated and properness).
10. Varieties as schemes.
11. Sheaf cohomology.
12. Cech cohomology.
13. Cohomology of P^n.
14. Serre duality and Riemann Roch for curves.

References:

1. Hartshorne, Algebraic geometry.
2. Stack Project, Online notes.
3. Qing Liu, Algebraic geometry and arithmetic curves.

Complex Geometry

Introduction: Complex Geometry is the study of complex manifolds and, more generally, complex analytic varieties. It involves algebraic as well as metric aspects. The subject is at the crossroads of algebraic geometry and differential geometry.

A complex manifold is a differentiable manifold endowed with the additional datum of a complex structure which is much more rigid than the geometrical structures in differential geometry. Due to this rigidity, one is often able to describe the geometry of complex manifolds more explicitly. The geometry also exhibits additional symmetries such as the Serre duality or the Hodge decomposition of cohomology.

The course will be very useful for those who want to seriously study or work in the areas of differential geometry, complex (algebraic) geometry, symplectic geometry, etc.

Contents: Holomorphic functions of several complex variables, complex and Hermitian structures, complex manifolds, projective spaces, holomorphic vector bundles, divisors and line bundles, differential Calculus on complex manifolds, Kahler manifolds and Kahler identities, Hodge theory, Lefshetz theorems, Hermitian vector bundles and Serre duality, connections, curvature, Chern classes, Hirzebruch-Riemann-Roch theorem, Kodaira vanishing theorem, Kodaira embedding theorem.

References:

1) Complex Geometry: An Introduction by Daniel Huybrechts, Springer.
2) Principles of Algebraic Geometry by Griffiths and Harris, Wiley.

ANALYTIC NUMBER THEORY

(1) Arithmetic functions:Arithmetic functions, Multiplicative functions, M ¨obius inversion formula and appli- cations, Dirichlet series, Orders of arithmetical functions, Average orders of arithmetical functions

(2) Primes in arithmetic progressions: Summation techniques, Characters mod q, Dirichlet's theorem for primes in arithmetic progressions, Dirichlet's hyperbola method

(3) Prime number theorem:Chebyshev's theorem, Non vanishing of Dirichlet series on $Re(s) = 1$, The Ikehara- Wiener theorem

(4) The method of contour integration: Basic integrals related to Dirichlet series, Perron's formula, error terms in the prime number theorem

(5) Functional equations:Poisson summation formula, Analytic continuation of the Riemann zeta function, Gauss sums, Dirichlet L-functions

(6) Hadamard products: Jensen's theorem, Entire functions of order 1, Gamma function, Infinite products for $\xi(s)$, $\xi(s, \chi)$, Zero-free regions for $\zeta(s)$, $L(s, \chi)$

(7) Explicit formulas:Counting zeros, Explicit formulas for $\psi(x)$, Weil's explicit formula

(8) The Siegel-Walfisz Theorem:Siegel's theorem on lower bounds for Dirichlet L-values at $s = 1$, distribution of primes in arithmetic progression.

(9) Selberg class: The Phragm´en-Lindel¨of theorem, Basic properties of Selberg class functions, Selberg's conjectures.

(10) The class number formula:Dedekind zeta function for number fields, the analytic class number formula for qua- dratic and cyclotomic fields.

References:

(1) "Problems in analytic number theory" by M. Ram Murty, Graduate Texts in

Mathe- matics (206), Springer.

(2) “Analytic number theory” by Henryk Iwaniec and Emmanuel Kowalski, Colloquium Publications (Vol. 53), American mathematical Society.

(3) “A Course in Analytic Number Theory” by Marius Overholt, Graduate Studies in Math- ematics (Book 160), American Mathematical Society.

<u>**Lie Theory**</u>

- (Real) Lie groups Basic examples GL(n), SL(n), SP(2n), O(n) and their Lie algebras gl(n), sl(n), sp(2n), so(n)
- Ideals, homomorphism, Solvable Lie algebra, Simple Lie algebra, Nilpotent Lie algebras, Engel's Theorem-Characterization of nilpotent Lie algebra, Lie's theorem - Characterization of solvable Lie algebra, Cartan's criteria, Killing form, Cartan decomposition of a semisimple Lie algebra,
- Root systems of classical Lie algebras, Dynkin diagrams, Classification of root systems
- Weyl group of a root system, Representation theory of semisimple Lie algebras, highest weight theory, Representations of SL(2), sl(2) and sl(2,C), SU(2) and Weyl's unitary trick
- More possible topics (up to instructor's discretion): Chevalley groups, Structure theory and Highest weight theory for compact Lie groups, Verma Modules, Complex semisimple Lie algebras

Texts:

- Introduction to Lie algebra and representation theory by J. Humphreys, Springer GTM
- Representations of Compact Lie Groups by T. Bröcker & T. tom Dieck, Springer GTM

Low dimensional Topology

Some deep open problems in this field have been resolved in the last decade, like the Geometrization Theorem of Perelman and Thurston, which has resulted in the influx of several new ideas and techniques. The beauty of 3-manifolds is that the categories of piece-wise linear, smooth and topological 3-manifolds are isomorphic and so we can use techniques that are purely combinatorial, from differential/Riemannian geometry or entirely topological. This course will give students an opportunity to begin with the basics and reach a level from where they can start studying the current research material in this field.

Topics:

2 dimensional manifolds:Classification of surfaces, Covering/Branched covering spaces, Homotopy/Isotopy on surfaces - Dehn twist generators, Nielsen-Thurston's classification, Curve complex, Measured laminations for surfaces/Teichmuller space, Uniformization theorem, Hyperbolic geometry of surfaces.

3 dimensional manifolds:Topology of 3-manifolds: Schoenflies Theorem, Triangulations and Normal surface theory, Prime decomposition, Incompressible surfaces, Loop and Sphere Theorem, Haken Hierarchy, Heegaard splittings, Foliations and Laminations. Geometry of 3- manifolds: Seifert Fibered spaces, Hyperbolic 3-manifolds, Eight Thurston geometries, JSJ decomposition, Geometrization Theorem.

Knots and Links: Wirtinger presentation of knot groups, Thurston's classification of knots, Fundamental theorem of Lickorish and Wallace, Introduction to quantum invariants.

Reference:

- Introduction to 3-manifolds by Schultens.
- Knots and Links by Rolfsen.
- Lectures on Hyperbolic Geometry by Benedetti and Petronio.3-manifolds by Hempel.
- Lectures on Three-manifold Topology by Jaco.

- Geometries of 3-manifolds by Scott (survey article).
- Automorphisms of Surfaces after Nielsen and Thurston by Casson and Bleiler.
- Notes on basic 3-manifold topology by Hatcher.
- An introduction to knot theory by Lickorish.

<u>**Parameterized Algorithms**</u>

Many problems we want to solve are often NP-hard or worse, but somehow, they need to get solved anyway. What can we do? Over the years, multiple paradigms for coping with NP-hardness have been introduced: for instance, approximation algorithms, average-case analysis, and randomized algorithms were all borne out of a desire to solve hard problems by relaxing the problem or strengthening the model.

Within the last 20 years, a new paradigm has been introduced, where one measures the time complexity of an algorithm not just in terms of the *input length* but also a small side *parameter*. A priori, the parameter can be anything, but the interesting case is when complex instances of the problem still have relatively small parameter values. The overall goal is to identify interesting parameterizations of hard problems where we can design algorithms running in time polynomial in the input length but possibly exponential (or worse) in the small parameter. Such an algorithm is called "fixed-parameter tractable" and it is the gold standard for parameterized algorithms.

This is a subject which is still very new and fresh. The course will be heavily research-oriented with lots of open problems.

Contents: Kernelization: vertex cover, feedback arc set in tournaments, edge clique cover, crown decomposition, Expansion lemma, Kernels based on linear programming; sunflower lemma; Bounded Search trees: vertex cover, feedback vertex set, closest string; Randomized methods in parameterized algorithms: coloring coding for longest path, divide and conquer for longest path, derandomization; Treewidth: path and tree decompositions, dynamic programming on graphs of bounded treewidth, weighted independent set, dominating set; Fixed parameter Intractability: parameterized reductions, problems at least as hard as cliques, The W-hierarchy, problems complete for W[1] and W[2].

References:

1. Marek Cygan, Fedor V. Fomin, Lukasz Kowalik, Daniel Lokshtanov, Daniel Marx, Marcin Pilipczuk, Michal Pilipczuk and Saket Saurabh, Parametrized Algorithms, Springer, 2016.

2. Downey, R.G., Fellows, M.R., Parameterized Complexity, Springer, 1999.

PDE

Sobolev spaces: Weak derivative, definition of Sobolev space, properties, global approxima- tion by smooth functions, extension, embedding theorems, Rellich's compactness theorem, trace theorems, Poincare inequality, characterisation of H^{-1} space.

Second order elliptic equations: Formulation of Dirichlet and Neumann derivative problem, weak formulation, Lax-Milgram lemma, existence of weak solution, energy estimates, regu- larity upto the boundary, maximum principle, Eigenvalues and Eigenfunctions of symmetric elliptic operator.

Second order parabolic equations: existence of weak solution, regularity, maximum principles.

Second order hyperbolic equations: existence of weak solution, regularity, propagation of disturbance.

Semigroup theory: definition, elementary properties.

References

1. L. C. Evans–Partial Differential Equations

2. Kesavan– Topics in Functional Analysis and Applications

3. Haim Brezis–Functional Analysis, Sobolev Space and Partial Differential Equations

4. Adams–Sobolev Spaces

Representation Theory

Definitions, Basic examples, Subrepresentations, Irreducible representations, Tensor product of two representations, Symmetric square and alternating square, Character theory, The character of a representation, Schur's lemma; basic applications, Orthogonality relations for characters, Decomposition of the regular representation, Number of irreducible representations, Canonical decomposition of a representation, Explicit decomposition of a representation,

Invariant measure on a compact group, Linear representations of compact groups

Decomposition, of C[G], The center of C [G]

Induced representations; Mackey's criterion, Induction, The character of an induced representation, the reciprocity formula, Restriction to subgroups, Mackey's irreducibility criterion, Examples of induced representations, Artin's theorem, Brauer's theorem, standard applications of Brauer's theorem.

Textbook: Linear representations of finite groups, Serre

Several complex variables

Holomorphic functions: The homogeneous Cauchy-Riemann equations, Cauchy's integral formula, sequences and compactness in spaces of holomorphic functions, Montel's theorem, power series, domains of convergence, Reinhardt domains, Identity theorem, open mapping theorem, zero sets of holomorphic functions, Riemann removable singularity theorem, Rado's theorem, analytic sets.
Analytic continuation phenomena: Extension by means of Cauchy's integral formula, Laurent series, extension by power series, The inhomogenous Cauchy-Riemann equations, solution for compactly supported data, Hartog's phenomenon.
Domains of holomorphy and pseudoconvexity: Natural boundaries, Hartogs' pseudoconvexity, differentiable boundaries and Levi pseudoconvexity, plurisubharmonic functions and pseudoconvexity, holomorphic convexity, characterizations of domains of holomorphy.
Automorphisms of bounded domains: Cartan's uniqueness theorem, automorphisms of circular domains, Poincare's theorem.

Reference:

1. S. G. Krantz, Function theory of several complex variables
2. R. Narashimhan, Several complex variables
3. R. M. Range, Holomorphic functions and integral representations in several complex variables

Coding Theory

Codes: basic properties, minimum distance, covering radius, automorphisms and equivalence
Linear codes: Basic definitions and properties. Generator and Parity Check matrices.
Basic theory of Finite Fields
The MacWilliams identities, Dual codes
MDS codes, Reed Solomon codes, Perfect Codes
Bounds on the size of codes: Hamming, Singleton, Plotkin, Gilbert-Varshamov, Elias
Shannon-Hilbert entropy function.

Reference Books

1. Introduction to Coding theory, J. H. van Lint, Springer Graduate Texts in Mathematics vol. 86, third edition.
2. Fundamentals of Error Correcting Codes, Cary Huffmann and Vera Pless, Cambridge University Press.
3. Introduction to Coding theory, Ron Roth, Cambridge University Press.

Linear codes: Basic definitions, properties and problems. Basic theory of Finite Fields. The projective geometric point of view of linear codes. The MacWilliams identity and Weight enumerators. Construction and properties of special codes and families of codes like MDS codes and Reed Solomon codes. Structure of finite fields in more detail. A flavour of Algebraic Geometry codes.

1. Fundamentals of Error-Correcting Codes: C. Huffman and V. Pless (2010) Cambridge

2. Introduction to Coding Theory: Ron Roth (2006) Cambridge
3. The Theory of Error-Correcting Codes (North-Holland Mathematical Library): MacWilliams and Sloane (1977) North Holland Publishing Co.

Topics in Harmonic Analysis

- The Fourier series and Fourier transform, Basic properties, Cesáro and Abel summability, Pointwise convergence of the Fourier series, the Poisson summation formula.
- Applications to linear dispersive PDEs, e.g. Schrödinger equation, Airy equation, Korteweg-de Vries (KdV) equations. Oscillatory integrals and time Decay estimates.
- Interpolation of Operators:
    - The Riesz-Thorin Convexity Theorem, Applications to Young and Hausdorff-Young inequalities.
    - Marcinkiewicz Interpolation Theorem, Applications to Hardy-Littlewood maximal function and Hardy-Littlewood Sobolev inequalities
    - The Stein Interpolation Theorem, Hörmander-Mikhlin multiplier theorem
- Littlewood–Paley theory. Applications to maximal operator, singular integral operator and almost orthogonality principle in L^p.

References:

1. E.M. Stein and R. Shakarchi, Fourier Analysis. An Introduction. Princeton University Press, 2011.
2. E.M. Stein and G. Weiss. Introduction to Fourier Analysis on Euclidean Spaces, Princeton Univ. Press, Princeton N.J. 1971.
3. E. Stein, Singular integrals and differentiability properties of functions. Princeton Mathematical Series, 30. Princeton University Press, 1970.
4. F. Linares and G. Ponce, Introduction to Nonlinear Dispersive Equations, Springer, 2014.
5. Loukas Grafakos, Classical and Modern Fourier Analysis
6. Hajer Bahouri, Jean-Yves Chemin, and Raphaël Danchin, Fourier Analysis and Nonlinear Partial Differential Equations
7. G.B. Folland, Real Analysis: Modern Techniques and Their Applications, Second Edition