# MTH201: EXTRA PROBLEMS FOR WEEK 1

(1) What is the maximum number of non-pivot and non-zero entries possible for a RREF matrix of size $m \times n$ and rank $r$ with pivot sequence $(p_1, p_2, \dots, p_r)$.

For which pivot sequence (with $r$ fixed), is the above number maximum/minimum ?

(2) Given constants $a$ and $b$, for which scalars $c$ is the vector $(1, c, c^2)^t$ a linear combination of the vectors $(1, a, a^2)^t$ and $(1, b, b^2)^t$.
(Recall a linear combinations of vectors $\vec{u}$ and $\vec{v}$ is a vector of the form $a\vec{u} + b\vec{v}$ where $a$ and $b$ are scalars.)

(3) Find the polynomial $p(t) = a + bt + ct^2 + dt^3$ whose graph goes through the points $(0,1), (1,0), (-1,0)$ and $(2,-15)$. Your solution must involve bringing a $4 \times 5$ augmented matrix into RREF.

4-) Do the following T/F problems with reasoning.



**38.** If $A$ and $B$ are any two $n \times n$ matrices of rank $n$, then $A$ can be transformed into $B$ by means of elementary row operations.

**39.** If a vector $\vec{v}$ in $\mathbb{R}^4$ is a linear combination of $\vec{u}$ and $\vec{w}$, and if $A$ is a $5 \times 4$ matrix, then $A\vec{v}$ must be a linear combination of $A\vec{u}$ and $A\vec{w}$.

**40.** If matrix $E$ is in reduced row-echelon form, and if we omit a row of $E$, then the remaining matrix must be in reduced row-echelon form as well.

**41.** The linear system $A\vec{x} = \vec{b}$ is consistent if (and only if) $\operatorname{rank}(A) = \operatorname{rank}\left[A \mid \vec{b}\right]$.

**42.** If $A$ is a $3 \times 4$ matrix of rank 3, then the system $A\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ must have infinitely many solutions.

**43.** If two matrices $A$ and $B$ have the same reduced row-echelon form, then the equations $A\vec{x} = \vec{0}$ and $B\vec{x} = \vec{0}$ must have the same solutions.

**44.** If matrix $E$ is in reduced row-echelon form, and if we omit a column of $E$, then the remaining matrix must be in reduced row-echelon form as well.

**45.** If $A$ and $B$ are two $2 \times 2$ matrices such that the equations $A\vec{x} = \vec{0}$ and $B\vec{x} = \vec{0}$ have the same solutions, then $\operatorname{rref}(A)$ must be equal to $\operatorname{rref}(B)$.

**46.** A lower triangular $3 \times 3$ matrix has rank 3 if (and only if) the product of its diagonal entries is nonzero.

**47.** If $ad - bc \neq 0$, then the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ must have rank 2.

**48.** If vector $\vec{w}$ is a linear combination of $\vec{u}$ and $\vec{v}$, then $\vec{u} + \vec{v} + \vec{w}$ must be a linear combination of $\vec{u}$ and $\vec{u} + \vec{v}$.

**49.** If the linear system $A\vec{x} = \vec{b}$ has a unique solution and the linear system $A\vec{x} = \vec{c}$ is consistent, then the linear system $A\vec{x} = \vec{b} + \vec{c}$ must have a unique solution.

**50.** A matrix is called a 0–1-matrix if all of its entries are ones and zeros. True or false: The majority of the 0–1-matrices of size $3 \times 3$ have rank 3.