TRUE OR FALSE? *Do the even numbered problems*

1. The polynomials of degree less than 7 form a seven-dimensional subspace of the linear space of all polynomials.

2. The function $T(f) = 3f - 4f'$ from C^∞ to C^∞ is a linear transformation.

3. The lower triangular 2×2 matrices form a subspace of the space of all 2×2 matrices.

4. The kernel of a linear transformation is a subspace of the domain.

5. The space $\mathbb{R}^{2\times3}$ is five-dimensional.

6. If $f_1, \dots, f_n$ is a basis of a linear space V, then any element of V can be written as a linear combination of $f_1, \dots, f_n$.

7. The space P_1 is isomorphic to $\mathbb{C}$.

8. If the kernel of a linear transformation T from P_4 to P_4 is $\{0\}$, then T must be an isomorphism.

9. If W_1 and W_2 are subspaces of a linear space V, then the intersection $W_1 \cap W_2$ must be a subspace of V as well.

10. If T is a linear transformation from P_6 to $\mathbb{R}^{2\times2}$, then the kernel of T must be three-dimensional.

11. All bases of P_3 contain at least one polynomial of degree ≤ 2.

12. If T is an isomorphism, then T^{-1} must be an isomorphism as well.

13. The linear transformation $T(f) = f + f''$ from C^∞ to C^∞ is an isomorphism.

14. All linear transformations from P_3 to $\mathbb{R}^{2\times2}$ are isomorphisms.

15. If T is a linear transformation from V to V, then the intersection of $\text{im}(T)$ and $\ker(T)$ must be $\{0\}$.

16. The space of all upper triangular 4×4 matrices is isomorphic to the space of all lower triangular 4×4 matrices.

17. Every polynomial of degree 3 can be expressed as a linear combination of the polynomial $(t-3)$, $(t-3)^2$, and $(t-3)^3$.

18. If a linear space V can be spanned by 10 elements, then the dimension of V must be ≤ 10.

19. The function $T(M) = \det(M)$ from $\mathbb{R}^{2\times2}$ to $\mathbb{R}$ is a linear transformation.

20. There exists a 2×2 matrix A such that the space V of all matrices commuting with A is one-dimensional.

21. The linear transformation $T(M) = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} M$ from $\mathbb{R}^{2\times2}$ to $\mathbb{R}^{2\times2}$ has rank 1.

22. If the matrix of a linear transformation T (with respect to some basis) is $\begin{bmatrix} 3 & 5 \\ 0 & 4 \end{bmatrix}$, then there must exist a nonzero element f in the domain of T such that $T(f) = 3f$.