MTH 201 – ASSIGNMENT 4

(1) Consider a matrix A and let $B = \text{RREF}(A)$.
   a) Is ker(A) necessarily equal to ker(B)? Explain.
   b) Is Im(A) necessarily equal to Im(B)? Explain.
(2) Find a nontrivial linear relation among the vectors $(1,2)^t, (2,3)^t, (3,4)^t$.
(3) Consider the vectors $v_1, v_2, \ldots, v_m$ in $\mathbb{R}^n$ with $v_m = \vec{0}$. Are these vectors linearly independent?
(4) Consider two subspaces V and W of $\mathbb{R}^n$.
   a) Is the intersection $V \cap W$ necessarily a subspace of $\mathbb{R}^n$?
   b) Is the union $V \cup W$ necessarily a subspace of $\mathbb{R}^n$?
(5) Find a basis for the subspace of $\mathbb{R}^3$ defined by $2x_1 + 3x_2 + x_3 = 0$.
(6) Consider a 5×4 matrix $A = [v_1\, v_2\, v_3\, v_4]$. We are told that the vector $(1,2,3,4)^t$ is in $\text{Ker}(A)$. Express v_4 as a linear combination of v_1, v_2, v_3.
(7) Consider an $m \times n$ matrix A and an $n \times m$ matrix (with $n \neq m$) such that $AB = I_m$. (We say that A is a left inverse of B.) Are the columns of B linearly independent? What about the columns of A?
(8) Consider an $n \times p$ matrix A and a $p \times m$ matrix B. We are told that the columns of A and the columns of B are linearly independent. Are the columns of the product AB linearly independent as well?