MTH 201 – ASSIGNMENT 2

(1) Suppose matrix A is transformed into matrix B by a sequence of elementary row operations. Is there a sequence of elementary row operations that transforms B into A ? Explain your answer.

(2) Is there a sequence of elementary row operations that transforms

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \text{into} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}? \quad \text{Explain.}$$

(3) Consider the equations $\begin{vmatrix} x & +2y & +3z & = 4 \\ x & +ky & +4z & = 6 \\ x & +2y & +(k+2)z & = 6 \end{vmatrix}$, where k is an arbitrary constant.

a) For which values of the constant k does this system have a unique solution?
b) When is there no solution?
c) When are there infinitely many solutions?

(4) Consider a linear system of four equations with three unknowns. We are told that the system has a unique solution. What does the reduced row-echelon form of the coefficient matrix of this system look like? Explain your answer.

(5) **T/F:** If $A = [\vec{u}\,\vec{v}\,\vec{w}]$ and $\text{RREF}(A) = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$, then the equation $\vec{w} = 2\vec{u} + 3\vec{v}$ holds.

(6) Do the even numbered problems from 1-34 in the following List of T/F Questions. Supply reasons, not just T/F.

Chapter One Exercises

TRUE OR FALSE?[19]

Determine whether the statements that follow are true or false, and justify your answer.

1. If A is an $n \times n$ matrix and $\vec{x}$ is a vector in $\mathbb{R}^n$, then the product $A\vec{x}$ is a linear combination of the columns of matrix A.

[19] We will conclude each chapter (except for Chapter 9) with some true–false questions, over 400 in all. We will start with a group of about 10 straightforward statements that refer directly to definitions and theorems given in the chapter. Then there may be some computational exercises, and the remaining ones are more conceptual, calling for independent reasoning. In some chapters, a few of the problems toward the end can be quite challenging. Don't expect a balanced coverage of all the topics; some concepts are better suited for this kind of questioning than others.

2. If vector $\vec{u}$ is a linear combination of vectors $\vec{v}$ and $\vec{w}$, then we can write $\vec{u} = a\vec{v} + b\vec{w}$ for some scalars a and b.

3. Matrix $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is in reduced row-echelon form.

4. A system of four linear equations in three unknowns is always inconsistent.

5. There exists a 3×4 matrix with rank 4.

6. If A is a 3×4 matrix and vector $\vec{v}$ is in $\mathbb{R}^4$, then vector $A\vec{v}$ is in $\mathbb{R}^3$.

7. If the 4×4 matrix A has rank 4, then any linear system with coefficient matrix A will have a unique solution.

8. There exists a system of three linear equations with three unknowns that has exactly three solutions.

9. There exists a 5×5 matrix A of rank 4 such that the system $A\vec{x} = \vec{0}$ has only the solution $\vec{x} = \vec{0}$.

10. If matrix A is in reduced row-echelon form, then at least one of the entries in each column must be 1.

11. The system $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 0 & 0 \end{bmatrix} \vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is inconsistent.

12. There exists a 2×2 matrix A such that $A \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

13. If A is a nonzero matrix of the form $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then the rank of A must be 2.

14. $\operatorname{rank} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix} = 3$

15. The system $A\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ is inconsistent for all 4×3 matrices A.

16. There exists a 2×2 matrix A such that

$$A \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \text{and} \quad A \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

17. $\operatorname{rank} \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix} = 2$

18. $\begin{bmatrix} 11 & 13 & 15 \\ 17 & 19 & 21 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 13 \\ 19 \\ 21 \end{bmatrix}$

19. There exists a matrix A such that $A \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}$.

20. Vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is a linear combination of vectors

$$\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}.$$

21. If the system $A\vec{x} = \vec{b}$ has a unique solution, then A must be a square matrix.

22. If A is any 4×3 matrix, then there exists a vector $\vec{b}$ in $\mathbb{R}^4$ such that the system $A\vec{x} = \vec{b}$ is inconsistent.

23. There exist scalars a and b such that matrix

$$\begin{bmatrix} 0 & 1 & a \\ -1 & 0 & b \\ -a & -b & 0 \end{bmatrix}$$

has rank 3.

24. If $\vec{v}$ and $\vec{w}$ are vectors in $\mathbb{R}^4$, then $\vec{v}$ must be a linear combination of $\vec{v}$ and $\vec{w}$.

25. If $\vec{u}$, $\vec{v}$, and $\vec{w}$ are nonzero vectors in $\mathbb{R}^2$, then $\vec{w}$ must be a linear combination of $\vec{u}$ and $\vec{v}$.

26. If $\vec{v}$ and $\vec{w}$ are vectors in $\mathbb{R}^4$, then the zero vector in $\mathbb{R}^4$ must be a linear combination of $\vec{v}$ and $\vec{w}$.

27. If A and B are any two 3×3 matrices of rank 2, then A can be transformed into B by means of elementary row operations.

28. If vector $\vec{u}$ is a linear combination of vectors $\vec{v}$ and $\vec{w}$, and $\vec{v}$ is a linear combination of vectors $\vec{p}$, $\vec{q}$, and $\vec{r}$, then $\vec{u}$ must be a linear combination of $\vec{p}$, $\vec{q}$, $\vec{r}$, and $\vec{w}$.

29. A linear system with fewer unknowns than equations must have infinitely many solutions or none.

30. The rank of any upper triangular matrix is the number of nonzero entries on its diagonal.

31. There exists a 4×3 matrix A of rank 3 such that $A \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \vec{0}$.

32. The system $A\vec{x} = \vec{b}$ is inconsistent if (and only if) $\operatorname{rref}(A)$ contains a row of zeros.

33. If A is a 4×3 matrix of rank 3 and $A\vec{v} = A\vec{w}$ for two vectors $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^3$, then vectors $\vec{v}$ and $\vec{w}$ must be equal.

34. If A is a 4×4 matrix and the system $A\vec{x} = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$ has a unique solution, then the system $A\vec{x} = \vec{0}$ has only the solution $\vec{x} = \vec{0}$.

35. If vector $\vec{u}$ is a linear combination of vectors $\vec{v}$ and $\vec{w}$, then $\vec{w}$ must be a linear combination of $\vec{u}$ and $\vec{v}$.

36. If $A = \begin{bmatrix} \vec{u} & \vec{v} & \vec{w} \end{bmatrix}$ and $\operatorname{rref}(A) = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$, then the equation $\vec{w} = 2\vec{u} + 3\vec{v}$ must hold.

37. If A and B are matrices of the same size, then the formula $\operatorname{rank}(A + B) = \operatorname{rank}(A) + \operatorname{rank}(B)$ must hold.