MTH 201 - ASSIGNMENT 1

(1) Solve by Gaussian elimination:
$$\begin{vmatrix} x_1 & -7x_2 & +x_5 & = & 3 \\ & x_3 & -2x_5 & = & 2 \\ & x_4 & +x_5 & = & 1 \end{vmatrix}.$$
(2)
$$\begin{vmatrix} x_4 & +2x_5 & -x_6 & = & 2 \\ x_1 & +2x_2 & +x_5 & -x_6 & = & 0 \\ x_1 & +2x_2 & +2x_3 & -x_5 & +x_6 & = & 2 \end{vmatrix}$$
: Solve by Gaussian elimination: (3) The reduced row-echelon forms of the augmented matrices of three systems as

(2)
$$\begin{vmatrix} x_4 & +2x_5 & -x_6 & = & 2 \\ x_1 & +2x_2 & & +x_5 & -x_6 & = & 0 \\ x_1 & +2x_2 & +2x_3 & & -x_5 & +x_6 & = & 2 \end{vmatrix}$$
: Solve by Gaussian elimination:

(3) The reduced row-echelon forms of the augmented matrices of three systems are given below. How many solutions does each system have?

$$a) \begin{bmatrix} \begin{smallmatrix} 1 & 0 & 2 & | & 0 \\ 0 & 1 & 3 & | & 0 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \qquad b) \begin{bmatrix} \begin{smallmatrix} 1 & 0 & | & 5 \\ 0 & 1 & | & 6 \end{bmatrix} \qquad c) \begin{bmatrix} \begin{smallmatrix} 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 3 \end{bmatrix}.$$

(4) Find the rank of the following matrices using Gaussian elimination.

$$a) \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \qquad b) \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad c) \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}.$$

(5) Which of the following matrices are in RREF?

$$a) \begin{bmatrix} 1 & 2 & 0 & 2 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad b) \begin{bmatrix} 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad c) \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix} \qquad d) \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \end{bmatrix}.$$

- (6) Find all 4×1 RREF matrices.
- (7) For which values of a, b, c, d and e is $\begin{bmatrix} 0 & a & 2 & 1 & b \\ 0 & 0 & 0 & c & d \\ 0 & 0 & e & 0 & 0 \end{bmatrix}$ RREF?
- (8) How many pivot sequences are there for RREF matrices of size $2 \times 2, 3 \times 2, 2 \times 3$.