Indian Institute of Science Education and Research, Pune

MID Semester Examination, January 2019

Course name: Multivariable Calculus **Course code:** MTH 102
Date: February 20, 2019 *Duration:* 2 hours
Instructor: Krishna Kaipa Total points: 35

Instructions:

- This question paper consists of 4 questions and 1 printed page. Each subpart of each problem carries 4 marks except Question 3c) which carries 3 marks.
- It is recommended that you re-check your work for calculation mistakes.
- No need to simplify your expressions.

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1. a) Find the projection of $\vec{U} = -\vec{i} + \vec{j} + \vec{k}$ onto $\vec{V} = 2\vec{i} + \vec{j} - 3\vec{k}$.

   b) Let $\vec{u}, \vec{v}$ be nonzero vectors in $\mathbb{R}^n$. Under what conditions does equality hold in the Cauchy-Schwarz inequality $|\vec{u} \cdot \vec{v}| \leq ||\vec{u}||||\vec{v}||$? Give proper mathematical reasoning. You are <u>not</u> allowed to use the notion of the angle θ between u and v.

2. a) Find an equation for the plane in $\mathbb{R}^3$ passing through $(2, -1, 3), (0, 0, 5)$ and $(5, 7, -1)$.

   b) Find the distance to the point $(6, 1, 0)$ from the plane through the origin that is perpendicular to $\vec{i} - 2\vec{j} + \vec{k}$.

3. a) Find the directional derivative of $f(x, y, z) = xyz$ in any direction (unit vector) which is normal to the surface $yx^2 + xy^2 + yz^2 - 3 = 0$ at $(1, 1, 1)$.

   b) Let n, R be constants. Consider the ideal gas law $PV = nRT$ where V, T and P are the volume, temperature, and the pressure. You may assume that each of P, T, V is a function of the remaining variables. Calculate $\partial V/\partial T, \partial T/\partial P, \partial P/\partial V$ and show that their product equals -1.

   c) Repeat part b) with the gas law of a van der Waals gas given by $P = \frac{RT}{V-\beta} - \frac{\alpha}{V^2}$, where α, β, R are some constants.

4. a) Find all second partial derivatives of $f(x, y) = \sec^3(4y - 3x)$.

   b) Let $F(x, y, z) = (e^{xz}, \sin(xy), x^5y^3z^2)$. Find the divergence of F and the curl of F.