

Duration: 2 hrs Total Marks 30

Solve any six of following eight questions; they carry equal marks.

- 1. Assuming that the orbits in the Bohr model contain an integer number of de Broglie wavelengths, show that the orbital angular momentum must be quantised, and that the energy levels follow the $1/n^2$ pattern. You can borrow ideas from classical mechanics in this hybrid model.
- 2. The hyperfine splitting in the hydrogen atom is given by

$$\Delta \nu = A \left[\frac{8\pi}{3} |\psi(0)|^2 + \left\langle \frac{3\cos^2 \theta - 1}{r^3} \right\rangle \right],$$

where A is a constant. Calculate $\Delta \nu$ for the 2p state.

- 3. Which of the 2p and the 2s electrons in the helium atom experiences lesser screening of the nuclear charge due to the 1s electron? Explain why.
- 4. What, approximately, is the wavelength (in nm) of the Balmer-alpha-like transition, $1s3p \rightarrow 1s2s$, in helium, assuming that the inner electron is always in the 1s orbit and screens the outer electron from the nuclear charge.
- 5. Write down the electronic configurations of the Be and B atoms (Z = 4, 5) in their first excited states along with the respective term symbol ${}^{2S+1}[L]_{r}$.
- 6. A hydrogen atom is in the state

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$$\Psi = a\,\psi_{100} + b\,\psi_{211} + c\,\psi_{210} + d\,\psi_{21-1},$$

where a, b, c, d are real numbers. Photons of energy 12 eV are incident on a collection of such atoms. Will there be emission of electrons? If yes, what are the possible energies? If not, what would be the energies of the H atoms upon absorption of photons?

- 7. An emission line from a star is observed to have certain spectral distribution, which is approximated by a Gaussian function, with the best fit values of the centroid and the standard deviation being 486.1 nm and 0.2 nm, respectively. What is the estimate of the temperature of the star, if the emission line is from a level with a lifetime of 12 ns?
- 8. Consider a two-level atom with stimulated emission and absorption rates given by

$$R_{21} = B_{21}N_2u(\omega)a(\omega)$$
$$R_{12} = B_{12}N_1u(\omega)a(\omega),$$

where $a(\omega)$ is the natural lineshape (i.e. the lineshape is not a δ -function). If the lifetime of the upper state is τ , show that the gain for a laser oscillator based on these two levels is given by

$$g(\omega) = (N_2 - N_1) \frac{\pi c^2}{2\omega^2 \tau} a(\omega)$$

Obtain the dimensions (units) of the lineshape function using this relationship.