Some Notes Regarding Experiments

Excerpts from Notes by Ramana Athreya and others

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Safety and Security in the Lab

Whenever you work in your lab, you should be aware of the possible dangers and hazards. This lab has relatively low level of dangers or hazards inherent to the apparatus. Nonetheless, there are dangers all around us, and we must be aware of them and try our best to prevent accidents.

- Accidents can be simple ones: such as cutting yourself by a blade or puncturing your muscle by a wire or a pin, or dropping something heavy on your foot. So, it is vital to pay attention to everything you do, no matter how small the task is.
- It is important to learn the correct use of all equipments, tools, and instruments. Incorrect use of even very ordinary instruments can lead to serious injury.
- Read carefully the manuals provided with the apparatus. This will prevent inadvertent damage to the instrument as well as improve the safety of the user.
- Electrical hazards (shock, short circuit) can spring quite easily if proper procedures are not followed. Use insulated tools where necessary. Switch off power when devices are not in use. Switch power on in the proper sequence. Be cautious about high currents or high voltages.
- Improper use or poor maintenance of electric outlets/equipment can lead to short circuit and fire. Be vigilant! Make sure you know where the mains switchboard or contact breakers are in the lab. *Turn them off in case of an accident,or if you suspect something is going wrong. But do this only if you can do this without taking undue risks, otherwise leave it alone.*
- Make sure you know where the fire extinguishers are located and read the instructions on them right away. Don't wait for a fire to start before you find out about them.
- Know where the escape routes are. In case there is an accident, fire, etc, leave the lab by the escape route *rapidly, but in an orderly manner, without rushing, without pushing each other, and without panic.*

A few remarks on performing experiments

1. Read the write-up on the experiment before you start on it. Since the lab is usually open have a look at the experimental set up as well.

2. Identify the derived and measured (independent) parameters.

3. Employ the largest possible range of the independent parameter in your experiment.

4. Have an idea of how errors propagate from the measured parameter to the derived parameter — the higher the power index in the relationship between the two, the faster the errors magnify while going from one to the other.

5. Always make more than one measurement of a parameter — typically 5–7 is a good number. Multiple measurements give you a more accurate representative value as well as an estimate of its accuracy.

6. Graphs are meant to be visual aids to analysis — make sure that they do aid! They should be neat, with the axes well labelled and marked with fiducial numbers. Make full use of the graph space — do not confine the plot to a tiny portion of the graph.

7. A computer printout of the graph is preferred though not mandatory.

8. If more than one measurement has been made at a value either show all the points on the graph or plot the mean and the standard error … or plot both if they do not clutter the graph too much. Error bars should be shown even on individual points if they are available.

9. Plot the expected theoretical curve for reference on the same graph where appropriate and possible.

10. In general when two plots on different graphs are to be compared make sure that both are plotted to the same scale, and if practical, the same range.

11. Put some thought into plotting the right quantities in a graph — they need not be either the measured or the derived quantities. The plotted variables may be some modified version of the two. Ease of subsequent analysis and conveying the meaning should be the guiding principle behind each graph.

12. Where one needs to estimate parameters from the graph plot transformed versions of the measured and derived quantities such the plot is a straight line and the required estimate is either the slope of the line or one of the intercepts.

13. Overplot a smooth curve on the points scatter — Do not join the dots! This smooth curve should be based on physics (the theoretical reference curve) and not driven by a spreadsheet's choice of parameters.

14. Every measurement or estimate will have an associated error. It is absolutely essential to quote the same. Note that the error may not be the same as the discrepancy between the estimate of a parameter and its true value
    Error: uncertainty in the measurement or estimate of a parameter
    Discrepancy: Difference between the measurement/estimate and the true value
    If we measure g = 8.9 $\pm$0.2 m s^{-2}, the uncertainty or error is 0.2 whereas the discrepancy is $9.8 - 8.9 =$ 0.9 m s^{-2}. A discrepancy much larger than the measurement error suggests systematic errors (also called bias) in the experiment.

    It should be noted that the term *Error* is loosely used to mean either measurement/estimation error or discrepancy (as defined above). In this course students should be careful in their usage and must abide by the above.

Statistical Analysis

Multiple measurements of a physical quantity rarely result in a single value ... because of 2 reasons:

1. The quantity itself may be multi-valued ... heights of Indians
2. No measurement process has infinite accuracy ... temperature fluctuations, human error, errors in fiducial markings, instrumental malfunction, etc.

There are two kinds of measurement errors:

- systematic errors (bias): this results in the measured value always being offset by a fixed amount from the true value ... e.g. a weighing machine with a spring which has undergone plastic deformation, or a scale in which each "millimeter" interval is only 0.99 millimetre
- random errors: this results in a scatter of measured values on either side of the true value. In general, both kinds of errors will occur in an experiment though one or the other may dominate. Furthermore, the magnitude of the two errors can vary in different regions of the parameter space ... e.g. a deformed spring will result in different systematic errors for objects of different masses.

Therefore, in the real world one is forced to deal with a distribution of values and in an experiment we have to make multiple measurements ... the more the better. Consequently, from these multiple measurements one has to extract a representative value and a dispersion. Statistics is a technique for determining the number of measurements to make, extract representative single values from a distribution, and estimate the accuracy of the representative value. Furthermore, since resources are finite and one cannot measure every single value of a distribution (think of the heights of Indians), statistics is also used to translate the representative value from a (sub-) sample to the true value for the whole population. In general, the sample value will be different from the true population value. Statistics tells us how to design our experiment to make this difference as little as is required to reach some conclusion ... e.g. Are Indians taller than the French ?

We will discuss 3 different groups of statistical parameters here:

1. Representative values: mean, median, mode
2. Dispersion: variance and standard deviation (aka root mean square deviation — rms)
3. Standard error: error on the representative value

Mean (aka average or arithmetic mean) is the most common representative parameter used for a distribution. The mean for a variable **X**, observed through **n** measurements $\mathbf{x}_i$, is denoted $\langle X \rangle$ or $\bar{X}$

$$\bar{x} = \frac{\Sigma_{i=1}^{n} w_i x_i}{\Sigma_{i=1}^{n} w_i}$$

where, w_i is the weight of each x_i. If the weights are all equal then we obtain the more commonly used expression

$$\bar{x} = \frac{\Sigma_{i=1}^{n} x_i}{n}$$

Median is the central value (or the average of two central values) when the measurements are all arranged in an ascending (or descending) sequence.

Mode is the value with the highest frequency of occurrence.
No one parameter is better than the others in all situations:

- Mean: is the most representative central value for a good distribution.
- Median: is a more stable central value in the presence of outliers or when the number of measurements (n) is small.
- Mode: is always guaranteed to be identical to one of the measurements, whereas neither the mean nor the median may be equal to one of the measurements. But, the mode can be at an extreme of the distribution.

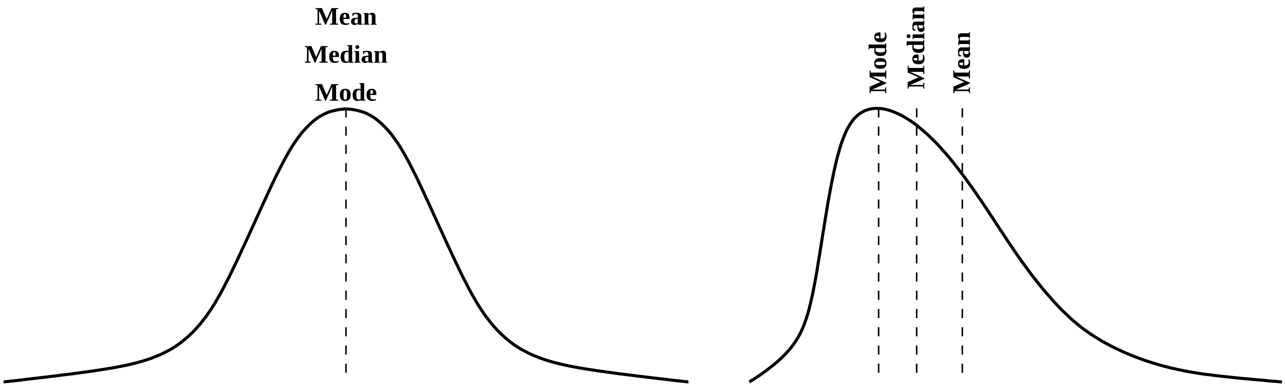

Figure 1: The behaviour of mean, median and mode for symmetric and asymmetric distributions. The asymmetry may either be intrinsic to the parameter, or it can be due to a large number of spurious (error) values. The mean is best for a smooth and well-behaved distribution, but it can be significantly affected by large outliers.

Standard Deviation

The standard deviation, or the root mean square (RMS) deviation of the parameter **X** is given by

$$\sigma_x = \sqrt{\frac{\Sigma_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{n}{n-1}\left(\overline{x^2} - \bar{x}^2\right)} = \sqrt{V_x}$$

where, V_x is the variance of the distribution. Note that the denominator is $n-1$ and not n.

A narrower distribution will have a smaller standard distribution. Further, in the case of a standard curve the probability of occurrence of a value is defined given the mean and the standard deviation (see Fig. 2) In the presence of noise approximately 68% will lie within 1σ of the mean, 95% within 2σ and 99.74% within 3 σ. The probability that a measurement falls outside 3σ due to statistical fluctuations due to error is only 0.26%. Therefore, any measurement value deviating from the expected (or mean) value by more than 3σ is unlikely to be due to measurement error and hence suggests that the model from which the expected value was derived was incorrect.

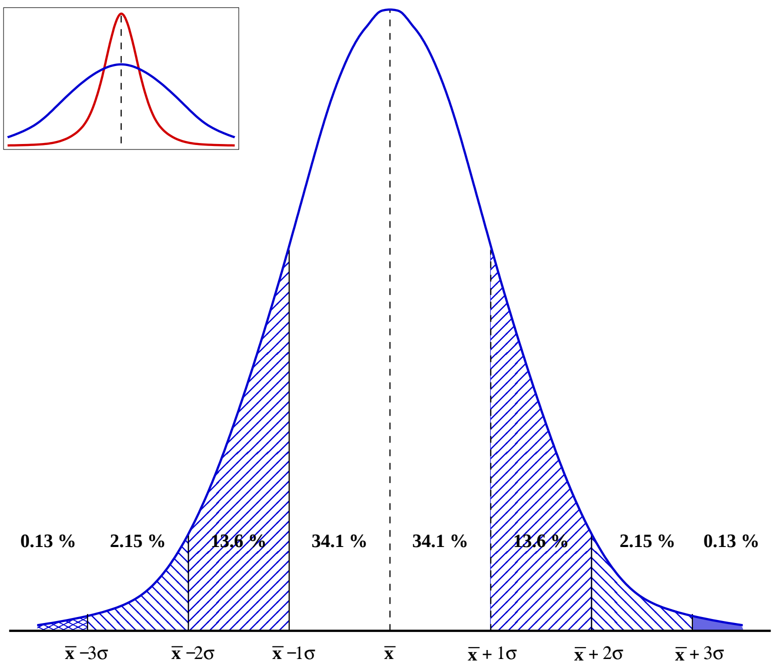

Figure 2: Gaussian or Normal distribution. Inset: A narrower distribution will have a smaller distribution than a broader one. The main figure shows the integral probability of the distribution within the corresponding region.

Standard Error: the error on the mean

It should be obvious that randomly selected sub-samples from a population will in general have means which differ from each other. In general, this difference is related to the standard deviation of the parent sample and the number of measurements in the sub-sample Usually, the mean values of the various sub-samples themselves form a distribution whose standard deviation is equal to the standard deviation of the parent population divided by the square-root of the number of measurements in each subsample. Therefore the error on the mean value is given by

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$$

A measured value has no meaning unless it is accompanied by its error. In text a measurement is usually written as [mean $\pm$ std.error]. Sometimes the error quoted is thrice the above value (for a 3σ range).

On a graph the mean values are plotted as points while the error on each are plotted as a vertical bar, whose length indicates either the standard error or three times its value (see Fig. 3). When a theoretical model differs from a measured value by much more than its error bar it means either that the data is wildly wrong, or more usually that the theoretical model on which the prediction is based is inappropriate. In Fig. 3, the straight-line fit dos not fit all the data and therefore only has a limited range of validity. On the other hand the non-linear (dashed) curve fits the data over the entire range. Usually, deviations from a simple model occur at one or the other extreme of the range of independent parameter values. Therefore, it is important to explore as large a range of the parameter space as possible.

The error on the particular value of a variable may be determined either by making multiple measurements and determining the standard error or by an estimate of various errors in the experimental set up. If the error model is correct the measured error should match the estimated error. Alternatively, one can make measurements of a known phenomenon and estimate the error from the scatter around the model curve (Fig. 4).

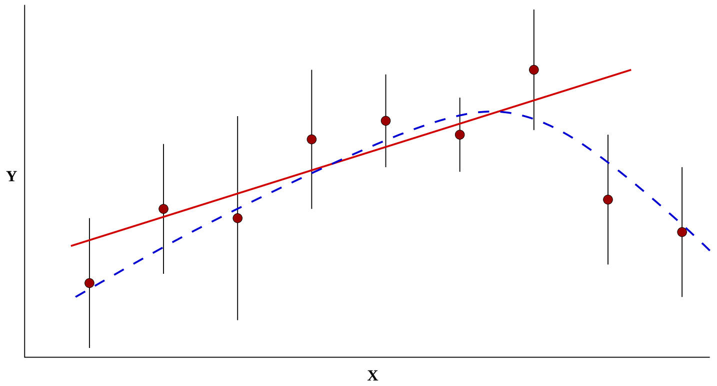

Figure 3: Error bars on data. The error bars indicate the range within which the *true* value is likely to be given the measured value. Any theoretical model seeking to explain measurements must fall within the error bars. In this figure the straight line fit (solid line) is only valid range of the data, while the non-linear curve (dashed line) fits all the points.

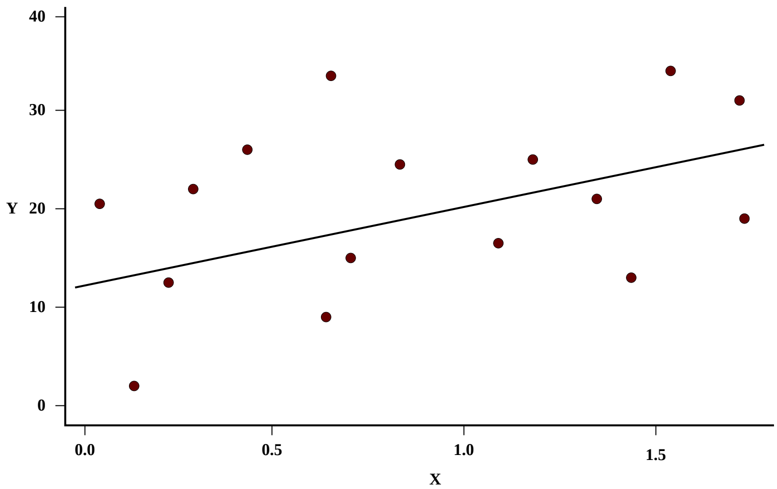

Figure 4: A known model (solid line) along with measured values. Under the assumption that the errors are the same for all measurements one can estimate the error by measuring the deviation of the points from the model curve.

Error Propagation

In general the quantity that we wish to estimate is usually not directly measured but estimated from measurements of other parameters; e.g. we measure velocity by measuring the distance travelled in some time interval. Therefore, any errors in the measurement of both distance and time will contribute to the error in the velocity estimate. This cascading of errors is termed error propagation. If the desired secondary quantity (X) is a function of one or more primary (or secondary) quantities A, B, C

$$X = f(A, B, C, \ldots)$$

then

$$\Delta X^2 = \left(\frac{\delta f}{\delta A}\right)^2 (\Delta A)^2 + \left(\frac{\delta f}{\delta B}\right)^2 (\Delta B)^2 + \left(\frac{\delta f}{\delta C}\right)^2 (\Delta C)^2 + \ldots$$

where, ΔX is the error on the mean value of X, and so on. Since the standard error on X is a measure of the error in the mean value

$$\sigma_X^2 = \left(\frac{\delta f}{\delta A}\sigma_A\right)^2 + \left(\frac{\delta f}{\delta B}\sigma_B\right)^2 + \left(\frac{\delta f}{\delta C}\sigma_C\right)^2 + \ldots$$

This expression is true only if the errors in A, B C, ... are not correlated with each other. Correlated errors are beyond the scope of this course and their analysis will not be taken up here.

Accuracy and Precision

Accuracy and precision are two words you will often encounter in the context of any measurement. Accuracy describes how close the measured value of a parameter is to the standard or true value. *An accurate measurement, or an accurate device gives values close to true value.*

If a measurement is repeated several times, we expect the values to cluster around a certain value. Precision is a measure of how close to each other several measurements are; it is an indicator of the scatter in the data. High precision implies small spread or scatter of values.

The graph (Fig. 5) explains the difference between accuracy and precision very nicely.

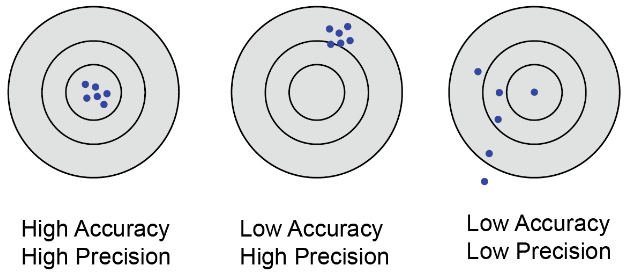

Figure 5: A measurement will result in a scatter of values. The extent of scatter represents the precision of the measurement, while the deviation of the mean of the measured values from the true, or reference, value represents its accuracy (Figure courtesy: Wikepedia)