Assignment 9 - Metric Spaces, completions

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Let (X, d) denote a metric space. A sequence $x_1, x_2, ...$ in X will be denoted by the notation (x_n). Let $\mathcal{C}[X]$ denote the set of all Cauchy sequences in X and $\tilde{X}$ denote the equivalence classes $\mathcal{C}[X]/\sim$ where $(x_n) \sim (y_n)$ if $\lim_{n\to\infty} d(x_n, y_n) = 0$. For a Cauchy sequence (x_n), let $[(x_n)] \in \tilde{X}$ denote its equivalence class. Let $\tilde{d}$ denote the metric on X where

$$\tilde{d}([(x_n)], [(y_n)]) := \lim_{n\to\infty} d(x_n, y_n).$$

Let $i : X \to \tilde{X}$ be the map which sends a point x to the class defined by the constant sequence $x, x, x, ...$

1. Show that if (x_n) is a Cauchy sequence in X, then $i(x_1), i(x_2), ...$ is a sequence in $\tilde{X}$ which converges to $[(x_n)]$.