Assignment 7 - Metric Spaces

MTH204, Spring 2017. IISER Pune.

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1. Let (X, d) be a metric space.
   (i) Show that arbitrary union of open subsets of X is open.
   (ii) Show that finite intersection of open subsets of X is open.
2. What are all the open and closet sets in a discrete metric space?
3. In $\mathbb{R}^2$ with usual metric, give examples of the following
   (i) Open set.
   (ii) Closed set.
   (iii) A set which is neither open nor closed.
   (iv) A set which is both open and closed.
4. What is the closure of the following subsets of $\mathbb{R}$.
   (i) $\mathbb{Z}$.
   (ii) $\mathbb{Q}$.
   (iii) $(-2, -1) \cup (0, \infty)$.
   (iv) any finite subset.
   (v) $\{1/n \mid n \in \mathbb{N}\}$.
5. Let X be a metric space and $A \subset X$. Let $\overline{A} := A \cup \{\text{all limit points of } A\}$. Show that $\overline{A}$ is a closed in X.
6. Let X be a metrix space, $A \subset X$ and $x \in X$. Show that x is a limit point of A iff there exists a sequence $x_1, x_2, ...$ in A which converges to x.
7. Give an example of a metric space X and subsets A and Y of X with $A \subset Y$, such that A is open in Y but not in X.